A Generalization of Credal Networks

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foundations of statistics

frequentist approach	likelihood approach	Bayesian approach
empirical	empirical	personalistic
repeated-sampling	conditional	conditional

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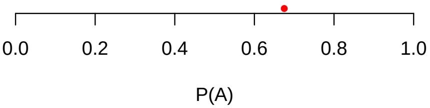
generalizations:

precise probability



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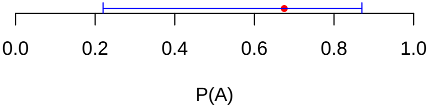
$\downarrow$

interval probability



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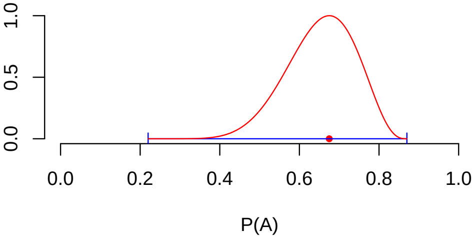
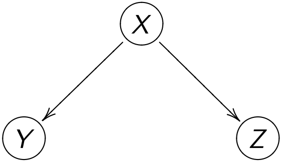
fuzzy probability



probabilistic graphical models

$X, Y, Z \in \{0, 1\}$

Y and Z independent conditional on X:



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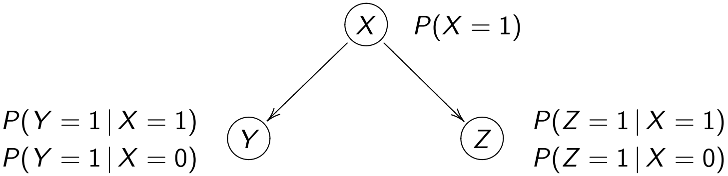

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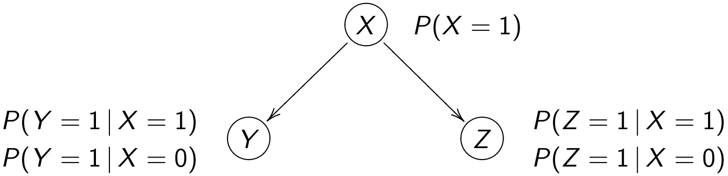

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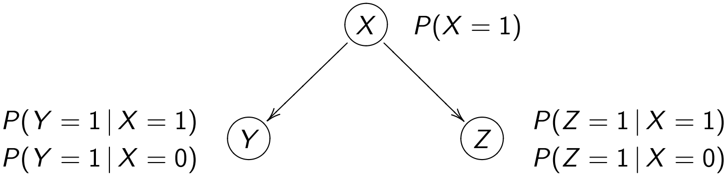

generalizations:

precise probabilities: Bayesian networks

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interval probabilities: credal networks

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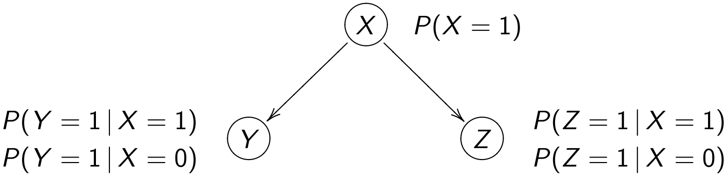

generalizations:

precise probabilities:	Bayesian networks
$\downarrow$	
interval probabilities:	credal networks
$\downarrow$	
fuzzy probabilities:	hierarchical networks

training data

X	Y	Z	#
0	0	0	21
0	0	1	6
0	1	0	30
0	1	1	7
1	0	0	9
1	0	1	15
1	1	0	5
1	1	1	7
			100

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simulated according to:

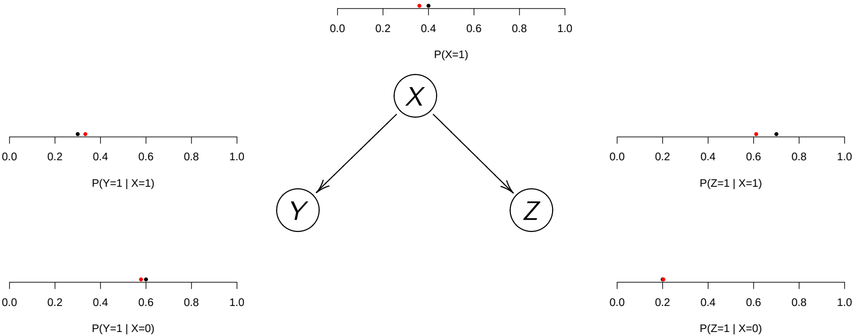
$$\begin{tikzcd} & X \arrow[dl] \arrow[dr] & \\ Y & & Z \end{tikzcd}$$

$P(X = 1) = 0.4$

$P(Y = 1 \mid X = 1) = 0.3$
$P(Y = 1 \mid X = 0) = 0.6$

$P(Z = 1 \mid X = 1) = 0.7$
$P(Z = 1 \mid X = 0) = 0.2$

Bayesian network via MLE



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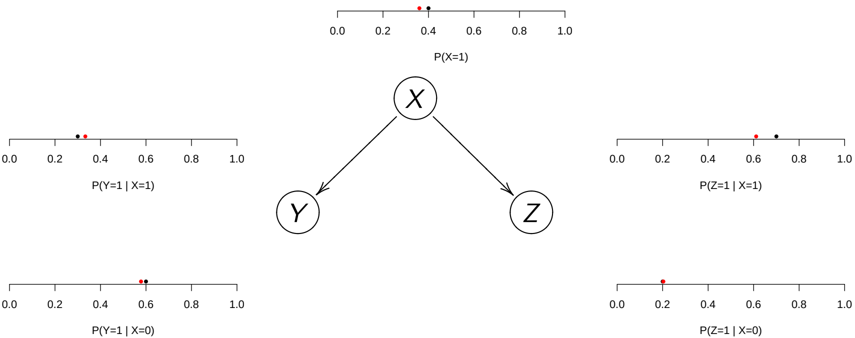
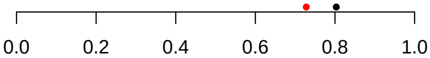
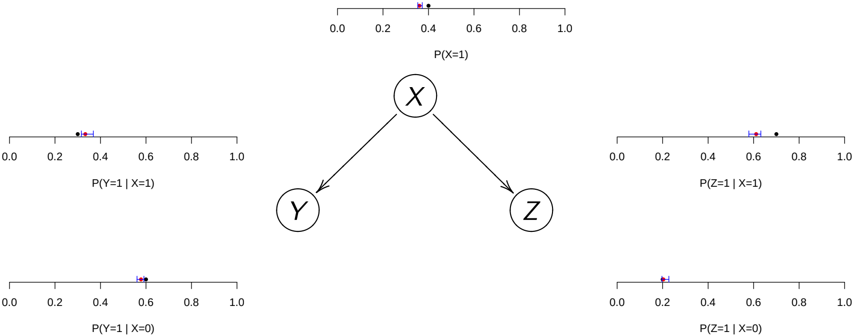


$\Rightarrow \; P(X = 1 \mid Y = 0,\, Z = 1)$:



credal network via IDM (with $s = 2$)



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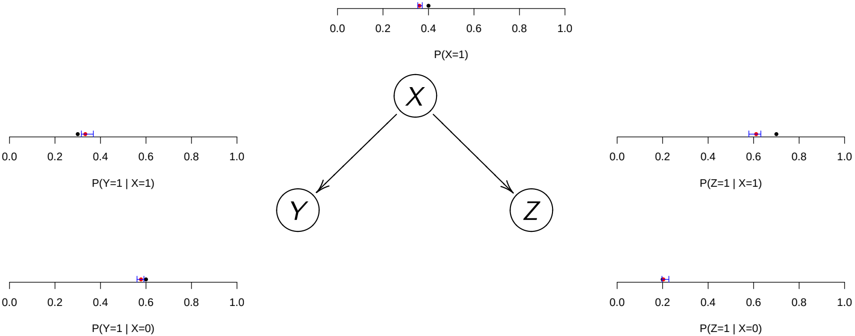

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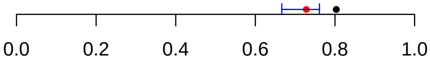
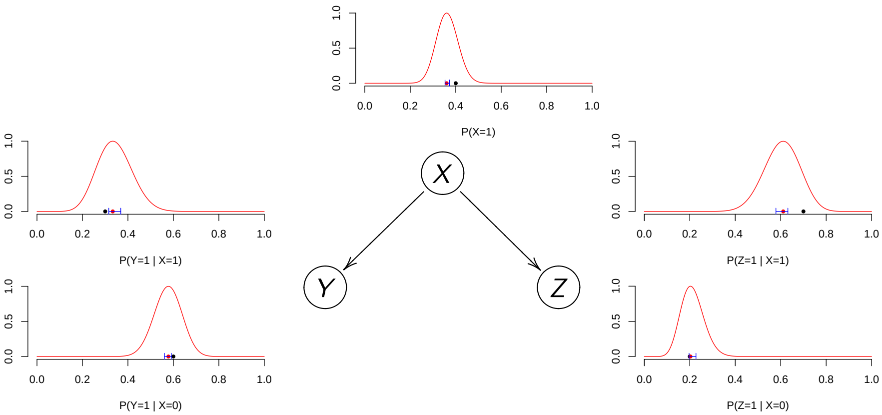


hierarchical network



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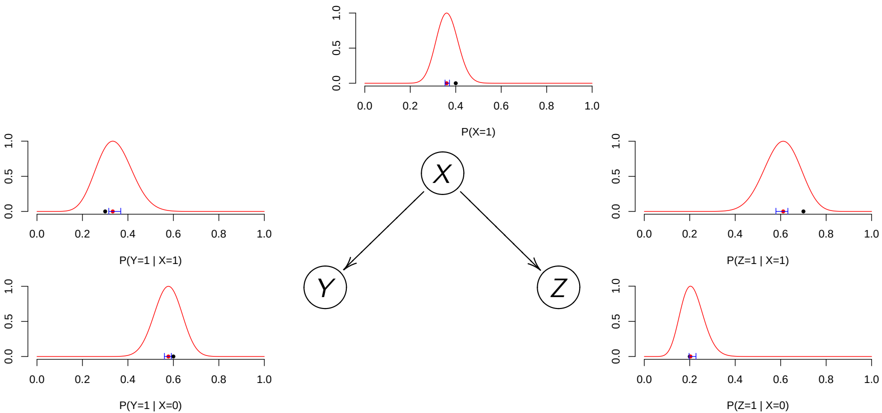

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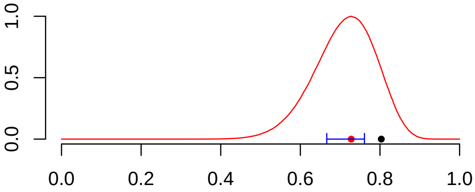
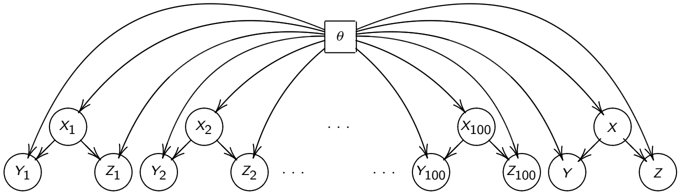
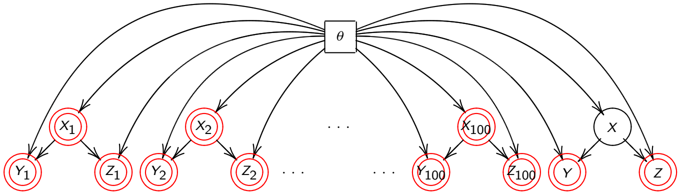


global models



prior ignorance
about $\theta \in [0, 1]^5$

global models



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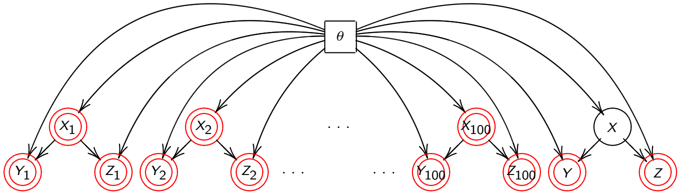
302 variables observed

global models



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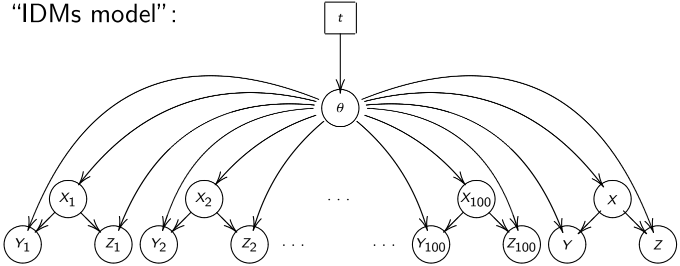

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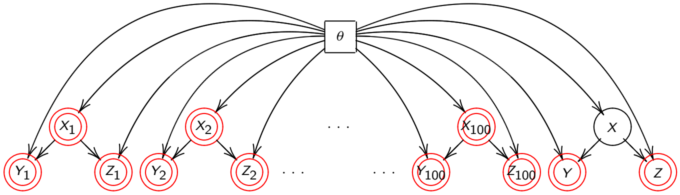

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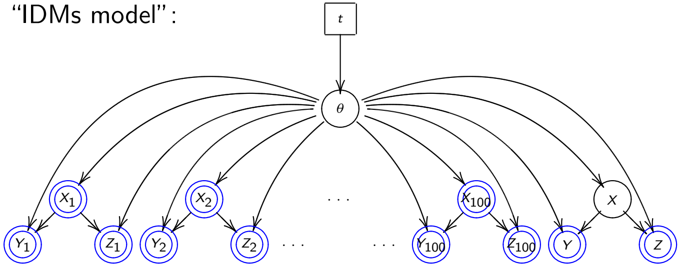

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conclusions and results

advantages of (likelihood-based) **fuzzy probability** over interval probability:

- **more expressive** (relative plausibility of different values in the probability interval)
- **more powerful updating rule** (extracts more information from the data)
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mathematical results of the paper:

- **d-separation** implies conditional irrelevance in hierarchical networks
- hierarchical networks can be described by **convex sets of measures**, and it suffices to consider the extreme points