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ALL IN A AND/MOUNT PROCESSION/AND ADDRESS Exchangeable lower previsions GERT DE COOMAN^{1,4}, BRIE QUARCHEREUR^{1,4,4} aut ENEIQUE MIRANDA⁷ "Gene Delevity, DEBH Rosseni Cong. Exfordigipael: Jerjannis VII, 902 Zerjannis & Jano Louisy, Delevitore Rossenic Cong. Exfordigipael: Jerjannis VII, 902 Zerjannis & Talaverity of Unide. Byo. of Institute and Question Rossenic Codes Senis, ab, 1987, Orian, Julie Kandi "genergingenetization". We record Place sequences of se-nature data (Inte-terms of complete 1. Introdu This same day annumed to be observed to det The first det 'equivalent' ex EXCHANCEABLE LOWER PREVINCING CERTOR COORDS, BRIC CONCREMENT, AND CORRECT MEADER 2008 writekt menufe discussion of 6 Do Pinetii pr sines, or lair pr able in specify Chapter 3: Th allow for a sh This paper deals with brind models for both linite and crossiable sequences of exchange able smalless variables taking a linite number of values. When such sequences of random do maline workshot taking a kuka mashen of scheme Views mis despenses of stability as a summal of scheme barrangehold, since the sources have been specific as the source of the strength of the state of the star star star strength of the state of the strength of the state of the strength of the state of the state of the state of the strength of the 1265 v1 110.761 0 (1996). Ver Hinry & Lanzaneg (2006), 2019 (2019) and 2019 (2019). A structure distance in distance in the interpretation of the sectory and the sectory of the sectory and the sectory of the sectory

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Exchangeable lower previsions



- (in)finite sequences of finite-valued random variables
- exchangeability assessment & sequence order permutations
- sample sequences, count vectors & frequency vectors
- representation theorems
- exchangeable natural extension

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Exchangeable lower previsions

Immediate prediction under exchangeability





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- families of exchangeable lower previsions
- coherent updated exchangeable lower previsions
- count vectors as sufficient statistics

Immediate prediction under exchangeability ...

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Exchangeable lower previsions

General context: experiments & gambles

A subject who is uncertain about the experiment's outcome. Considers $f \in \mathcal{T}(\omega)$ after the experiment's outputse is ω . $f(\omega)$ when the experiment's outputse is ω . $D = \{\omega, \omega'\}$ A sample / is desirable to the subject if he 0 f(m) () The subject's capital is changed by *f*(*w*).

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Coherent sets of desirable gambles

A subject's set of desirable gandales $\mathcal{A} \subseteq \mathcal{G}(\Omega)$ models his beliefs 10100 The set of desirable complete if is coherent It is accelerated galaxies in a content of the second sec

Sets of weakly desirable gamples The subject considers a gamble f in $S(\Omega)$ weakly desirable f. The subject observes, or considers the possibility of observing, an by adding any desirable contain to it, another desirable contain is even if of Ω .

The subject's set of weakly desirable gambles is

ert set of desirable gambles of satisfies the following properties: $(f, f), f \in \mathcal{G}(\Omega), X \ge 0$ WD1. If f < 0 then $f \notin \mathcal{D}_{\theta}$ [assiding partial loss] $\begin{array}{c} \text{WDL} & \mathcal{M} \subset \mathcal{O} \text{Sets} \ f \in \mathcal{G}_{2} \text{ [anothing particul space]} \\ \text{WDL} & \mathcal{M} \subset \mathcal{O} \text{Sets} \ f \in \mathcal{O}_{2} \text{ [society of particul space]} \\ \text{WDL} & \mathcal{M} \subset \mathcal{O}_{2} \text{ sets} \ f \in \mathcal{O}_{2} \text{ sets} \text{sing} \\ \text{WDL} & \mathcal{M} \subset \mathcal{O}_{2} \text{ sets} \ f \in \mathcal{O}_{2} \text{ sets} \text{sing} \\ \text{WDL} & \mathcal{M} \subset \mathcal{O}_{2} \text{ sets} \ f \in \mathcal{O}_{2} \text{ sets} \text{sing} \\ \text{WDL} & \mathcal{M} \cap \mathcal{M} \cap \mathcal{M} \text{ sets} \ f \in \mathcal{O}_{2} \text{ sets} \ f \in \mathcal{O}_{2} \\ \text{WDL} & \mathcal{M} \cap \mathcal{M} \cap \mathcal{M} \text{ sets} \ f \in \mathcal{O}_{2} \end{array}$

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Assessments & their natural extension An assessment can consid of a set $\mathcal{A} \subset \mathcal{B}(\Omega)$ considered desirable by the subject. The assessment of avoids non-positivity if the 🖉 d(u) intersection of cosi(of) and (C(D) is empty.

 $d(\alpha) = \operatorname{coni}(C(\beta)(\alpha))$

Exchangeability

The experiment consists of the observation of the value of a set of a subject assesses that Y X, are exchangeable this means The regimment contracts on the destination on the leader of a day queries X,..., X, of random variables for which X is the finite set. That for any gamble / and any permutation X, he finds exchanging of cookies values. So the cookies in a cookies of the set. If the finite set is a set of the The registion invariant space of all such exchange cambles is

> If P ... consider of weakly desirable-particles, then so does its conis at hall $\mathcal{D}_{P_{2}} = \operatorname{coni}(\mathcal{D}_{P_{2}}) = \operatorname{span}(\mathcal{D}_{P_{2}})$.

Exchangeable natural extension

The assessment of avoids non-positivity under exchangeability if The exchangeable natural extension of 14 is $\mathcal{S}_{\mathbf{m}}^{\mathbf{n}}(d') \coloneqq \mathcal{D}_{\mathbf{m}} + \mathcal{E}(d').$

If ω' avoids non-positivity under exchangeability, then $I_{\omega}^{\alpha}(\omega')$ is the smallest exchangeable observed set of desirable and set on the

Representation

A set of desirable samples R on R^{∞} is otherest and exchangeable. The subject observes the values $I = (I_1, I_2, ..., I_k)$ or the court

 $\mathcal{R} = (Mally^N)^{-1}(\mathcal{S}),$ and in that case this ${\mathscr S}$ is uniquely determined by $\mathcal{F} = \{ \mathbf{r} \in \mathcal{H}(\mathcal{F}^n) : \mathbf{T}^n(\mathbf{r}) \in \mathcal{H} \} = \mathbf{M}_0 \mathbf{D}^N(\mathcal{H}).$

Exchangeable natural extension &

The assessment $\mathcal{A} \subset \mathcal{X}^n$ avoid non-coalities under exchange A nice result $\operatorname{Mailly}^{H}(\mathcal{A}_{m}^{H}(\mathcal{A}')) = \mathcal{A}(\operatorname{Mailly}^{H}(\mathcal{A}')).$

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Representing updated models

sector $\dot{m} = T^{*}(i)$ in J^{*} of the first a variables X_{1}, \dots, X_{n} If .If is a coherent and exchangeable set of desirable gambles. If is is a coherent and exchangeable set of decisions gardees on X⁻², then the representation of the two - because of sufficiency-identical updated models he uses is 214-348-0214

This representation is not an updated model of the representation $\mathcal{S}[m=\{g(a+\cdot)\colon L_{a}g\in\mathcal{S}\},$

A (m) = [4] (m-4) which is awa when m 2 m.

Exchangeable previsions & representation

A lower prevision \underline{p} on $\Psi(X^{(q)})$ is conserved and exchangeable iff there is some coherent base prevision \underline{p} and $\Psi(F^{(q)})$ — its constraint previous that \underline{p} are available. It is that case \underline{p} is uniquely observational by $\underline{p}=\underline{p}$ - $\underline{p}^{(q)}$.



 coherent sets of (weakly) desirable gambles

- natural extension
- updating
- relationship with previsions

Specific context: finite sequences

of possible values. So the possibility $X' = \{ \bigcirc, \bigcirc \}, N = 3$ space D is X'' and $s = (s_1, \dots, s_d)$ 4 (0.0.0) Pe is the set of all permutations it of the index set (1......N).

The associated permutation of \mathcal{X}^{N} is defined by $(\mathbf{Z} \mathbf{x})_{0} = \mathbf{x}_{N, 0}$ It is lifted to a permutation S' of $H'(S'^{(k)})$ by letting S'f = f + E.

With every sequence at observations corresponds a court vector in - able if $\mathcal{D}_{\rm de} \subseteq \mathcal{D}_{\rm de}$ or equivalently if

The country map $T^{0}, \mathcal{X}^{n} \rightarrow \mathcal{X}^{0}$ maps $T^{1}(\Phi, \Phi, \Phi) = (1, 1)$ T of is allower and exchangeable from it in also permutative for T of is a balance of T > Q. The country map $T^{0}, \mathcal{X}^{n} \rightarrow \mathcal{X}^{0}$ maps $T^{1}(\Phi, \Phi, \Phi) = (1, 1)$ T of is a balance of T > Q. The country map $T^{0}, \mathcal{X}^{n} \rightarrow \mathcal{X}^{n}$ maps T (Φ, Φ, Φ) = (1, 1) T of is a balance of T > Q. The country map $T^{0}, \mathcal{X}^{n} \rightarrow \mathcal{X}^{n}$ maps T (Φ, Φ, Φ) = (1, 1)

Permuted sequences have the same quart vector: a permutation

[1,2] = ((0,0,0), (0,0,0), (0,0,0))

Moving between sequence gambles and count gambles



The projection of a sequence pantitie / onto a permutation invariant

 $\mathfrak{ss}^{n}(f) := \frac{1}{2} \Sigma_{n-2n} \mathfrak{s}^{n}(f = \Sigma_{n-2} \operatorname{Mulb}^{n}(f)\mathfrak{m})_{n-2}$ where its value on an invariant atom [w] is given by

 $Mally^{H}(f|m) := \frac{1}{m} \sum_{y \in M} f(y).$ The court gandle corresponding to the sequence gandle / is $MaHy^{K}(f) := MaHy^{K}(f)$.

The permutation invariant sequence pandle is a one to one corres-

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SYSTeMS Research Group & FUM Research Unit, Ghent University (Gert.deCooman.Erik.Ouaeohebeur)#UGent.be

Continuent on observing (or in, the subject mod

Updating sets of desirable gambles

 $\mathcal{R}|\mathcal{R} = \{f_{\theta} : I_{\theta}f \in \mathcal{R}\}.$

If IF is a coherent set of desirable conding on (2, then IF III is a

Coherent previsions & desirability

The lower prevision of a gamble / associated to a set of desirable

 $\mathcal{E}_{\pi}(f) := \sup\{\mu \in \mathbb{R} : f - \mu \in \mathcal{A}\}.$

succession converses and of decisative gambies. If each that $\underline{p} = \underline{p}_{ij} = \underline{p}_{ij}$

Coherent lower previsions are less expressive uncertainty models

aniugate upper prevision 7...(/)

Its conjugate upper prevision $\overline{P}_{\omega}(f)$ is equal to $-\underline{p}_{\omega}(-f).$ A lower prevision \underline{p} is subsect if here

Exchangeable previsions

 $\mathscr{R}[d:=\{f(d,\cdot): I_{|d|=-d} \circ f \in \mathscr{R}\},$

A lower prevision $\sum \alpha r \mathcal{A}(\mathcal{X}^{\prime p})$ is exchargeable if there is some as

Updating exchangeable models The subject observes the values $I = (I_1, I_2, ..., I_n)$ or the point vec for an e^{-it} of the field 4-variables, X_{i},\ldots,X_{i} this means observing the event $\{t\} \in \mathcal{X}^{n}$ of $[n] \times \mathcal{X}^{n}$ or $[n] \times \mathcal{X}^{n}$ or the event $\{t\} \times \mathcal{X}^{n}$ or $[n] \times \mathcal{X}^{n}$ we are interested in interested about the centralizing i=N-i variables.

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on \mathcal{X}^n , then $\mathcal{X}[i$ and $\mathcal{R}]$ is are otherwrit and exchangeable sets of decades and lies on \mathcal{X}^n .

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Specific context: finite sequences

guardian X_1, \dots, X_n in research of possibility $X' = \{ [0, 0], N = 3 \}$

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I 9 ... consider of weakly desirable gambles, then so does be con-With every sequence of observations corresponds a court vector in $-able t \ \mathcal{D}_{an} \subseteq \mathcal{D}_{an}$ or equivalently.

The counting map T^{K_1} $\mathcal{X}^{W} \rightarrow \mathcal{S}^{W}$ maps $T^{h_1}(\Phi, \Phi, \Phi) = (1, 2)$ If it is scherert and exchangeable then it is also permutable for

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Exchangeable natural extension &

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Representing updated models 214-368/0214

This representation is not an updated model of the representation

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Coherent lower previsions are tess expressive uncertainty models

Updating exchangeable models

The subject observes the values $I = (I_1, I_2, \dots, I_n)$ or the court vecfor any e^+ of the first is variables X_{e},\ldots,X_{e} this means observing the event $\{I\}\times X^+$ of $|A|\times X^+$ or $|A|\times X^+$, we are interested in interescent about the remaining s=X-s variables.

on $\mathcal{X}^n,$ then \mathcal{R} is and \mathcal{R} is any coherent and exchangeable sets of desirable candida on $\mathcal{R}^n.$

Exchangeable previsions

A lower prevision $\sum \alpha r \mathcal{A}(\mathcal{X}^{\prime q})$ is exchargeable if there is some as

 $\mathscr{R}[d:=\{f(d,\cdot)\colon I_{[d]=A}\circ f\in\mathscr{R}\},$

ante voner prevision P. (1)

A lower prevision 2 is sufferent 2 them

The subject observes the values $t=(t_1,t_2,\ldots,t_k)$ or the count vector $\dot{m} = T^{0}(\vec{x})$ in \mathcal{A}^{0} of the first is variables X_{1}, \dots, X_{n} . If it is a coherent and exchangeable set of desirable gambles. $\mathcal{R} = (\mathrm{Mally}^N)^{-1}(\mathcal{S}),$

 $\mathcal{F} = \{ \mathbf{r} \in \mathcal{H}(\mathcal{F}^n) : \mathbf{T}^n(\mathbf{r}) \in \mathcal{H} \} = \mathbf{M}_0 \mathbf{D}^N(\mathcal{H}).$

A nice result $\operatorname{Mailly}^{H}(\mathcal{A}_{m}^{H}(\mathcal{A}')) = \mathcal{A}(\operatorname{Mailly}^{H}(\mathcal{A}')).$

Exchangeable previsions & representation A lower prevision \underline{p} on $\Psi(X^{(q)})$ is conserved and exchangeable iff there is some coherent base prevision \underline{p} and $\Psi(F^{(q)})$ — its constraint previous that \underline{p} are available. It is that case \underline{p} is uniquely observational by $\underline{p}=\underline{p}$ - $\underline{p}^{(q)}$.

which is awa when m 2 m.



exchangeability assessment & weakly desirable gambles

- exchangeable natural extension
- updating exchangeable models
- relationship with exchangeable previsions

General context: experiments & gambles

A subject who is uncertain about the experiment's outcome Considers $f \in \mathcal{T}(\omega)$ after the experiment's outputse is ω . $f(\omega)$ when the experiment's outputse is ω . $D = \{\omega, \omega'\}$ A sample / is desirable to the subject it to 0 f(m) () The subject's capital is changed by *f*(*w*).

Coherent sets of desirable gambles

A subject's set of desirable samples $\mathcal{R} \subset \mathcal{R}(Q)$ models his beliefs. NC (0) The set of desirable complete if is coherent It is satisfies the following schematry requirements of $\phi_1, \phi_2 \in \mathcal{G}(\Omega), \lambda \geq 0$.

Pe is the set of all permutations it of the index set (1......N). The associated permutation of $\mathcal{X}^{(0)}$ is defined by $(\mathbf{Z} \mathbf{x})_{0} = \mathbf{x}_{\mathbf{z} | \mathbf{z}^{(0)}}$

Permuted sequences have the same quart vector: a permutation

[1,2] = ((0,0,0), (0,0,0), (0,0,0))

Moving between sequence gambles and

count gambles

The projection of a sequence pantitie / onto a permutation invariant

 $\mathfrak{ss}^{n}(f) := \frac{1}{2} \Sigma_{n-2n} \mathfrak{s}^{n}(f = \Sigma_{n-2} \operatorname{Mulb}^{n}(f)\mathfrak{m})_{n-2}$

 $MaHy^{H}(f|m) := \frac{1}{m} \sum_{y \in M} f(y).$

 $\operatorname{MaHy}^{n}(f) := \operatorname{MaHy}^{n}(f)$.

The court sandle corresponding to the sequence sandle (is

where its value on an invariant atom [w] is given by

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Sets of weakly desirable gamples The subject considers a gamble f in $S(\Omega)$ weakly desirable f. The subject observes, or considers the possibility of observing, an by adding any desirable contain to it, another desirable contain is even if of Ω .

The subject's set of weakly desirable gambles is

ert set of desirable gambles. IF satisfies the following properties: $(f, f), f \in \mathcal{G}(\Omega), X \ge 0$ WD1. If f < 0 then $f \notin \mathcal{D}_{\theta}$ [assiding partial loss] $\begin{array}{c} \mbox{WC:} \ 2/ < 0 \ \mbox{ten} \ / \ 2/ \ \mbox{[anishing partice away]} \\ \mbox{WC:} \ 2/ < 0 \ \mbox{ten} \ / \ 2/ \ \mbox{[anishing partice away]} \\ \mbox{WC:} \ 2/ < 0 \ \mbox{ten} \ / \ 2/ \ \mbox{[anishing partice away]} \\ \mbox{WC:} \ 2/ < 0 \ \mbox{ten} \ / \ 2/ \ \mbox{[anishing partice away]} \\ \mbox{WC:} \ 2/ < 0 \ \mbox{ten} \ / \ 2/ \ \mbox{[anishing partice away]} \\ \mbox{WC:} \ 2/ < 0 \ \mbox{ten} \ / \ 2/ \ \mbox{[anishing partice away]} \\ \mbox{WC:} \ 2/ < 0 \ \mbox{ten} \ \mbox{ten} \ \ 2/ \ \mbox{[anishing partice away]} \\ \mbox{WC:} \ 2/ < 0 \ \mbox{ten} \ \ 2/ \ \mbox{ten} \ \ 2/ \ \ \ 2/ \ \ \ 2/ \ \ \ 2/ \$

gandies in 16; (2)

Assessments & their natural extension

An assessment can conside of a set $\omega' \subset \mathcal{T}(\Omega)$ considered desirable by the subject. The assessment of avoids non-positivity if the 🖉 d(u) intersection of cosi(of) and (Q_(D) is empty $d(\alpha) = \operatorname{coni}(C(\beta)(\alpha))$

E of assids recreased into the c(or) is the smallest orthogen and

The experiment consists of the observation of the value of a set of a subject assesses that Y X, are exchangeable this means The regimment contracts on the destination on the leader of a day queries X,..., X, of random variables for which X is the finite set. That for any gamble / and any permutation X, he finds exchanging of cookies values. So the cookies in a cookies of the set. If the finite set is a set of the of possible values. So the possibility $X' = \{ \bigcirc, \bigcirc \}, N = 3$ space D is X'' and $s = (s_1, \dots, s_d)$

is at hall $\mathcal{D}_{P_{2}} = \operatorname{coni}(\mathcal{D}_{P_{2}}) = \operatorname{span}(\mathcal{D}_{P_{2}})$. It is little to a permutation X' of H'(X''') by letting $X' f = f \circ X$. With every sequence of observations corresponds a count vector in $Able \in \mathcal{D}_{de} \subseteq \mathcal{D}_{de}$ or equivalently.

The country map $T^1, X^{\infty} - , T^{\infty}$ maps $T^1(\Phi, \Phi, \Phi) = (1, 2)$ If X is schemed and excharged in the 1 is also percentation for X is a contemport of X = 0.

Exchangeable natural extension

The assessment of avoids non-positivity under exchangeability if The exchangeable natural extension of 14 is

Representation

A set of desirable gambles $\mathcal R$ on $\mathcal R^{\mathcal R}$ is subwort and exchangeable

 $\mathcal{R} = (\mathrm{Mally}^N)^{-1}(\mathcal{S}),$

 $\mathcal{F} = \{ \mathbf{r} \in \mathcal{H}(\mathcal{F}^n) : \mathbf{T}^n(\mathbf{r}) \in \mathcal{H} \} = \mathbf{M}_0 \mathbf{D}^N(\mathcal{H}).$

Exchangeable natural extension &

The assessment $\mathcal{A} \subset \mathcal{X}^n$ avoid non-coalities under exchange A nice result $\operatorname{Mailly}^{H}(\mathcal{A}_{m}^{H}(\mathcal{A}')) = \mathcal{A}(\operatorname{Mailly}^{H}(\mathcal{A}')).$

Representing updated models

The subject observes the values $t=(t_1,t_2,\ldots,t_k)$ or the count we can $\phi = T^{2}(1)$ in J^{2} of the first is variables $X_{1}, ..., X_{k}$ If it is a coherent and exchangeable set of desirable gambles. 214-368/0214

This representation is not an updated model of the representation $\mathcal{S}[m=\{g(a+\cdot)\colon L_{a}g\in\mathcal{S}\},$

1. (a) - [4] [a - 4]

which is awa when m 2 m.

Exchangeable previsions & representation

A lower prevision \underline{p} on $\Psi(X^{(q)})$ is conserved and exchangeable iff there is some coherent base prevision \underline{p} and $\Psi(F^{(q)})$ — its constraint previous that \underline{p} are available. It is that case \underline{p} is uniquely observational by $\underline{p}=\underline{p}$ - $\underline{p}^{(q)}$.



representation theorem

- exchangeable natural extension & representation
- representing updated exchangeable models
- relationship with representing previsions

The permutation invariant sequence pandle is a one to one corres-Gert de Cooman & Erik Quaeghebeur

SYSTeMS Research Group & FUM Research Unit, Ghent University (Gert.deCooman.Erik.Ouaeohebeur)#UGent.be

Specific context: finite sequences Exchangeability

The registion invariant space of all such exchange cambles is

 $\mathcal{S}_{\mathbf{m}}^{\mathbf{n}}(d') \coloneqq \mathcal{D}_{\mathbf{m}} + \mathcal{E}(d').$

E ω' avoids non-positivity under exchangeability, then $I^{\rm ext}_{\rm co}(\omega')$ is the smallest exchangeable observed set of desirable and the induction include

for any e^+ of the first is variables X_{e},\ldots,X_{e} this means observing the event $\{I\}\times X^+$ of $|A|\times X^+$ or $|A|\times X^+$, we are interested in interescent about the remaining s=X-s variables. $\mathscr{R}[d:=\{f(d,\cdot)\colon I_{[d]=A}\circ f\in\mathscr{R}\},$ Exchangeable previsions A lower prevision $\sum \alpha r \mathcal{A}(\mathcal{X}^{\prime p})$ is exchargeable if there is some as

Updating sets of desirable gambles

 $\mathcal{R}(\mathbf{R}) = \{f_{\mathbf{R}}, I_{\mathbf{R}} f \in \mathcal{R}\}$

If IF is a coherent set of desirable conding on (2, then IF III is a

Coherent previsions & desirability

The lower prevision of a gamble / associated to a set of desirable

 $\mathcal{E}_{\pi}(f) := \sup\{\mu \in \mathbb{R} : f - \mu \in \mathcal{A}\}.$

succession converses and of decisative gambies. If each that $\underline{p} = \underline{p}_{ij} = \underline{p}_{ij}$

Coherent lower previsions are less expressive uncertainty models

Updating exchangeable models

The subject observes the values $I = (I_1, I_2, ..., I_n)$ or the point vec

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ante voner prevision P. (1)

A lower prevision p is subsect if there



General context: experiments & gambles

A subject who is uncertain about the experiment's outcome. Considers $f \in \mathcal{T}(\omega)$ after the experiment's outputse is ω . $f(\omega)$ when the experiment's outputse is ω . $D = \{\omega, \omega'\}$ A sample / is desirable to the subject if he 0 f(m) () The subject's capital is changed by *f*(*w*).

Coherent sets of desirable gambles

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Specific context: finite sequences

queries X_1, \dots, X_n is sense. If possible values. So the possibility $X' = \{ egin{array}{c} 0, 0 \\ N = 3 \end{array} \}$

The associated permutation of $\mathcal{X}^{(0)}$ is defined by $(\mathbf{Z} \mathbf{x})_{0} = \mathbf{x}_{\mathbf{z} | \mathbf{z}^{(0)}}$

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Moving between sequence gambles and

count gambles

The projection of a sequence pantitie / onto a permutation invariant

 $\mathfrak{ss}^{n}(f) := \frac{1}{2} \Sigma_{n-2n} \mathfrak{s}^{n}(f = \Sigma_{n-2} \operatorname{Mulb}^{n}(f)\mathfrak{m})_{n-2}$

 $Mally^{H}(f|m) := \frac{1}{m} \sum_{y \in M} f(y).$

The court sandle corresponding to the sequence sample if is

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The permutation invariant sequence pandle is a one to one corres-

where its value on an invariant atom (w) is given by

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Sets of weakly desirable gamples The subject considers a gamble f in $S(\Omega)$ weakly desirable f. The subject observes, or considers the possibility of observing, an by adding any desirable contain to it, another desirable contain is even if of Ω .

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 $\begin{array}{c} & \mathsf{WD}: Sf \in \mathsf{Oren} f \in \mathcal{G}_{\mathsf{F}} \text{ [accepting particle part]} \\ & \mathsf{WD}: Sf \in \mathcal{G}_{\mathsf{F}} \text{ then } Sf \in \mathcal{G}_{\mathsf{F}} \text{ [basis for } S_{\mathsf{F}} \in \mathcal{G}_{\mathsf{F}} \text{ [basis for } S_{\mathsf{F}} \in \mathcal{G}_{\mathsf{F}} \text{ then } S_{\mathsf{F}} \in \mathcal{G}_{\mathsf{F}} \text{ the } S_{\mathsf{F}} = S_{\mathsf{F}} \text{ the } S_{\mathsf{F}} \text{ the } S_{\mathsf{F}} = S_{\mathsf{F}} \text{ the } S_{\mathsf{F}} \text{ the } S_{\mathsf{F}} = S_{\mathsf{F}} \text{ the } S_{\mathsf{F}}$

gandies in 16; (2)

Assessments & their natural extension

An assessment can conside of a set $\omega' \subset \mathcal{T}(\Omega)$ considered desirable by the subject. The assessment of avoids non-positivity if the 🖉 d(u) intersection of cosi(cir) and (C(D) is empty.

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Exchangeability

The experiment consists of the observation of the value of a set of a subject assesses that Y X, are exchangeable this means The regimment contracts on the destination on the leader of a day queries X,..., X, of random variables for which X is the finite set. That for any gamble / and any permutation X, he finds exchanging of cookies values. So the cookies in a cookies of the set. If the finite set is a set of the The registion invariant space of all such exchange cambles is

I 9 ... consider of weakly desirable gambles, then so does be conis at hall $\mathcal{D}_{P_{2}} = \operatorname{coni}(\mathcal{D}_{P_{2}}) = \operatorname{span}(\mathcal{D}_{P_{2}})$. With every sequence of observations corresponds a count vector in $Able \in \mathcal{D}_{de} \subseteq \mathcal{D}_{de}$ or equivalently.

The country map $T^{1}, X^{N} \cdots, T^{N}$ maps $T^{1}(\Phi, \Phi, \Phi) = (1, 2)$ If \vec{x} is sub-set if an exchapped in term is also permutative for X.

Exchangeable natural extension

The assessment of avoids non-positivity under exchangeability if The exchangeable natural extension of 14 is

If ω' avoids non-positivity under exchangeability, then $I_{\omega}^{\alpha}(\omega')$ is the smallest exchangeable observed set of desirable and set on the

 $\mathcal{F} = \{ \mathbf{r} \in \mathcal{H}(\mathcal{F}^n) : \mathbf{T}^n(\mathbf{r}) \in \mathcal{H} \} = \mathbf{M}_0 \mathbf{D}^N(\mathcal{H}).$

Exchangeable natural extension &

The assessment $\mathcal{A} \subset \mathcal{X}^{n}$ avoid non-costivity under each

If is is a coherent and exchangeable set of decisions gardees on X⁻², then the representation of the two - because of sufficiency-identical updated models he uses is This representation is nut an updated model of the representation

 $\mathcal{S}[m=\{g(a+\cdot)\colon L_{a}g\in\mathcal{S}\},$

which is awa when m 2 m.

A (m) = [4] (m-4) Exchangeable previsions & representation

A lower prevision \underline{p} on $\Psi(X^{(q)})$ is conserved and exchangeable iff there is some coherent base prevision \underline{p} and $\Psi(F^{(q)})$ — its constraint previous that \underline{p} are available. It is that case \underline{p} is uniquely observational by $\underline{p}=\underline{p}$ - $\underline{p}^{(q)}$.



Best definition of coherence?

Infinite exchangeable sequences?

Representation for infinite exchangeable sequences?

The subject observes the values $I = (I_1, I_2, ..., I_n)$ or the court vec for each $x\in T^*$ of the first d variables X_1,\dots,X_d for an each observing the exect $\{I\}\times \mathcal{X}^*$ or $[a]\times \mathcal{X}^*$ or $[a]\times \mathcal{X}^*$. We are interested in inferences about the remaining d=N-d variables. Continuent on observing (or in, the subject mo $\mathscr{R}[d:=\{f(d,\cdot)\colon I_{[d]=A}\circ f\in\mathscr{R}\},$ $\mathcal{R}[\phi := \{f(g,\cdot) : I_{(g) \in \mathcal{R}} \text{ and } g \in [\phi]\}.$

Updating sets of desirable gambles

updated set of desirable gambles, the subset of H(A) given by $\mathcal{R}(\mathbf{R}) = \{f_{\mathbf{R}}, I_{\mathbf{R}} f \in \mathcal{R}\}$

If IF is a coherent set of desirable conding on (2, then IF III is a

Coherent previsions & desirability

The lower prevision of a gamble / associated to a set of desirable

 $\mathcal{E}_{\mu}(f) := \sup\{\mu \in \mathbb{R} : f - \mu \in \mathcal{A}'\}.$

Its conjugate upper previous $\overline{P}_{\omega}(f)$ is equal to $-\underline{C}_{\omega}(-f)$. A lower previous f is subsect if there exists one consert set of centralize gambles. If each that $\underline{C}_{\omega}=\underline{C}_{\omega}(f)$

Coherent lower previsions are less expressive uncertainty models

Updating exchangeable models

on $\mathcal{X}^n,$ then $\mathcal{X}[i]$ and $\mathcal{X}[i]$ is are coherent and exchangeable sets of desirable gambles on $\mathcal{X}^n.$

Exchangeable previsions

A lower prevision $\sum \alpha r \mathcal{A}(\mathcal{X}^{\prime p})$ is exchargeable if there is some as

Representing updated models

The subject observes the values $\vec{s} = (\vec{s}_1, \vec{s}_2, ..., \vec{s}_k)$ or the court vector $\dot{m} = T^{0}(\vec{x})$ in \mathcal{A}^{0} of the first is variables X_{1}, \dots, X_{n} . If it is a coherent and exchangeable set of desirable gambles.

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 $\mathcal{S}_{\mathbf{m}}^{\mathbf{n}}(d') \coloneqq \mathcal{D}_{\mathbf{m}} + \mathcal{E}(d').$

Representation

A set of deviative gambles $\mathcal R$ on $\mathcal R^{\mathcal M}$ is coherent and exchangeable

 $\mathcal{R} = (\mathrm{Mally}^N)^{-1}(\mathcal{S}),$

A nice result $\operatorname{Mailly}^{H}(\mathcal{A}_{m}^{H}(\mathcal{A}')) = \mathcal{A}(\operatorname{Mailly}^{H}(\mathcal{A}')).$

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Gert de Cooman & Erik Quaeghebeur

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General context: experiments & gambles

A subject who is uncertain about the experiment's outcome. Considers $f \in \mathcal{T}(\omega)$ after the experiment's outputse is ω . $f(\omega)$ when the experiment's outputse is ω . $D = \{\omega, \omega'\}$ accepts the following transaction: (I) the actual outcome is indemnified, and (ii) the actual outcome is indemnified, and (ii) the actual outcome is indemnified by f(w). f(w) = f(w)

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Coherent sets of desirable gambles

A subject's set of desirable gandales $\mathcal{A} \subseteq \mathcal{G}(\Omega)$ models his beliefs NC*(Q) The set of desirable complete if is coherent It is a state of the balance of the DZ. 8 / > 8 Two / r . Placonstine cartial saint: 12

Specific context: finite sequences

of possible values. So the possibility $X' = \{ \bigcirc, \bigcirc \}, N = 3$ space D is X'' and $s = (s_1, \dots, s_d)$

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Permuted sequences have the same quart vector: a permutation

[L2] = {(0.0.0), (0.0.0), (0.0.0)}

Moving between sequence gambles and

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The projection of a sequence pantitie / onto a permutation invariant

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The permutation invariant sequence pandle is a one to one corres-

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Sets of weakly desirable gamples The subject considers a gamble f in $\theta(\Omega)$ weakly desirable I. The subject observes, or considers the possibility of observing, an by abding any desirable contain to it, another desirable contain e event if of Ω .

The subject's set of weakly desirable gambles is

The set of weakly desirable gambles \mathcal{P}_{ϕ} corresponding to a safer-

gandies in 16; (2).

Assessments & their natural extension

An assessment can consid of a set $\mathcal{A} \subset \mathcal{B}(\Omega)$ considered desirable by the subject. The assessment of possible non-possible y if the 🖉 d(of) the assessment of advancementary and (C(D) is empty.

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Exchangeability

The experiment consists of the observation of the value of a ser-Let imperiant contact or in destination to init active in a set queries X, ..., X, of random satisfies for which X is the finite set of cookies values, So the cookies values for which X is the finite set of cookies values, So the cookies values for an of cookies values. So the cookies values for an of cookies values for a set of the set of cookies values. So the cookies values for an of cookies values for a set of the set of cookies values for a set of the set of cookies values. So the cookies of the set of cookies values for a set of the set of cookies values for a set of the cookies of the set of cookies values for an of the set of cookies values cookies of the set of cookies values for a set of the cookies of the cookies of the set of the set of the cookies of the cookies of the cookies of the set of the set of the cookies of the cookies of the set of the cookies of the set of the set of the set of the cookies of the cookies of the set of the cookies of the set of the s The registion invariant space of all such exchange cambles is

If P ... consider of weakly desirable-particles, then so does its conitst hall $\mathcal{D}_{2_{2}} = \operatorname{coni}(\mathcal{D}_{2_{2}}) = \operatorname{span}(\mathcal{D}_{2_{2}})$ It is lifted to a permutation S' of $H'(S'^{(k)})$ by letting S'f = f + E. With every sequence of observations corresponds a court vector in $det \mathcal{D}_{de} \subseteq \mathcal{D}_{de}$ or equivalently (

The country map $T^{0}, \mathcal{X}^{n} \rightarrow \mathcal{X}^{0}$ maps $T^{1}(\Phi, \Phi, \Phi) = (1, 1)$ T of is allower and exchangeable from it in also permutative for T of is a balance of T > Q. The country map $T^{0}, \mathcal{X}^{n} \rightarrow \mathcal{X}^{0}$ maps $T^{1}(\Phi, \Phi, \Phi) = (1, 1)$ T of is a balance of T > Q. The country map $T^{0}, \mathcal{X}^{n} \rightarrow \mathcal{X}^{n}$ maps T (Φ, Φ, Φ) = (1, 1) T of is a balance of T > Q. The country map $T^{0}, \mathcal{X}^{n} \rightarrow \mathcal{X}^{n}$ maps T (Φ, Φ, Φ) = (1, 1)

Exchangeable natural extension

The assessment of avoids non-positivity under exchangeability if A lower prevision $\sum_{i=1}^{n} \psi(X^{i_i})$ is exchangeable if there is some as of + 9x, avoids non-costlying The exchangeable natural extension of 14 is

If ω' avoids non-positivity under exchangeability, then $I_{\omega}^{\alpha}(\omega')$ is the smallest exchangeable observed set of desirable and set on the

Representation

A set of desirable samples R on R^{∞} is otherest and exchangeable. The subject observes the values $I = (I_1, I_2, ..., I_k)$ or the court

and in that case this .7 is uniquely determined by $\mathscr{S} = \{ g \in \mathscr{G}(\mathscr{K}^n) \colon \operatorname{T}^n(g) \in \mathscr{B} \} = \operatorname{Multp}^S(\mathscr{B}).$

A nice result $\operatorname{Mailly}^{H}(\mathcal{A}^{H}_{\mathfrak{m}}(\mathcal{A}')) = \mathcal{A}(\operatorname{Mailly}^{H}(\mathcal{A}')).$

Representing updated models

sector $\phi = T^{*}(i)$ in T^{*} of the first is variables $\Sigma_{1}, ..., \Sigma_{r}$. If it is a coherent and exchangeable set of desirable gambles. If is is a coherent and exchangeable set of decision gambles, on X⁽²⁾, then the representation of the two – because of sufficiency– identical updated models he uses is 214-340-024

This representation is not an updated model of the representation $\mathcal{S}[m=\{g(a+\cdot)\colon L_{a}g\in\mathcal{S}\},$

A (m) = [4] (m-4) which is awa when m 2 m.

Exchangeable previsions & representation

A lower prevision \underline{p} on $\Psi(X^{(q)})$ is conserved and exchangeable iff there is some coherent base prevision \underline{p} and $\Psi(F^{(q)})$ — its constraint previous that \underline{p} are available. It is that case \underline{p} is uniquely observational by $\underline{p}=\underline{p}$ - $\underline{p}^{(q)}$.



See you at the poster!

Updating exchangeable models The subject observes the values $I = (I_1, I_2, \dots, I_n)$ or the court vecfor each e^{-i} of the first e variables X_{i}, \dots, X_{d} for some observing the event $\{x\} \times \mathcal{X}^{*}$ of $[a] \times \mathcal{X}^{*}$ of $[a] \times \mathcal{X}^{*}$. We are interested in inferences about the event $[a] \times \mathcal{X}^{*}$ of $[a] \times \mathcal{X}^{*}$. We are interested in inferences

Continuent on observing (or in, the subject models his $\mathscr{R}[d:=\{f(d,\cdot): I_{[d]=-d}\circ f\in \mathscr{R}\},$ $\mathcal{R}[\phi := \{f(g,\cdot) : I_{(g) \in \mathcal{R}} \text{ and } g \in [\phi]\}.$

Updating sets of desirable gambles

Contingent on observing #, the subject models his beliefs using an

updated set of desirable gambles, the subset of H(A) given by $\mathcal{R}|\mathcal{R} = \{f_{\theta} : I_{\theta}f \in \mathcal{R}\}.$

If it's a coherent set of desirable candles on (2, then it's it a coherent set of desirable-gambles on R.

 $\mathcal{E}_{\mu}(f) := \sup\{\mu \in \mathbb{R} : f - \mu \in \mathcal{A}'\}.$

on \mathcal{X}^n , then $\mathcal{X}[i$ and $\mathcal{R}]$ is are otherwrit and exchangeable sets of decades and lies on \mathcal{X}^n .

Exchangeable previsions

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Exchangeable natural extension &

The assessment $\mathcal{A} \subset \mathcal{X}^n$ avoid non-coalities under exchange



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Coherent previsions & desirability The lower prevision of a gamble / associated to a set of desirable

 $\mathcal{P}_{\rm d}$ is the closure of $\mathcal{R}_{\rm c}$ excluding

The conjugate apper previous $P_{\omega}(f)$ is equal to $-\underline{e}_{\omega}(-f)$. A lower previous f is subset of f have the previous $P_{\omega}(f)$ is equal to $-\underline{e}_{\omega}(-f)$. A lower previous f is subset of f have the previous f and f are the previous f are the previous f and f are the previous f are the previous f are the previous f are the previous f and f are the previous f a Coherent lower previsions are less expressive uncertainty models.