Exchangeability for sets of desirable gambles

Gert de Cooman & Erik Quaeghebeur

UNIVERSITEIT GENT

Gert
de Cooman

Erik
Quaeghebeur

UNIVERSITEIT
GENT

SYSTeMS

Filip
Hermans

Keivan
Shariatmadar

Gert
de Cooman

Erik
Quaeghebeur

UNIVERSITEIT GENT

SYSTeMS

FUM

Filip Hermans

Keivan Shariatmadar

Gert de Cooman

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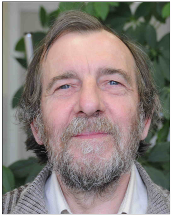

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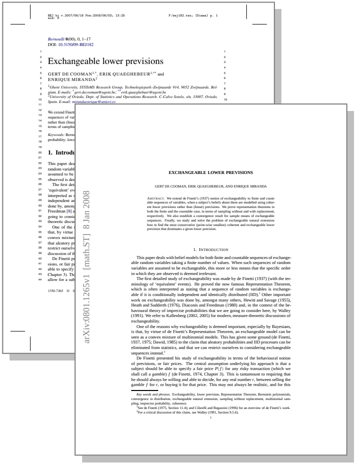

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Exchangeable lower previsions

GERT DE COOMAN[1,*], ERIK QUAEGHEBEUR[1,**] and
ENRIQUE MIRANDA[2]

[1]*Ghent University, SYSTeMS Research Group, Technologiepark–Zwijnaarde 914, 9052 Zwijnaarde, Belgium. E-mails:* [*]*gert.decooman@ugent.be;* [**]*erik.quaeghebeur@ugent.be*
[2]*University of Oviedo, Dept. of Statistics and Operations Research. C-Calvo Sotelo, s/n, 33007, Oviedo, Spain. E-mail: mirandaenrique@uniovi.es*

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EXCHANGEABLE LOWER PREVISIONS

GERT DE COOMAN, ERIK QUAEGHEBEUR, AND ENRIQUE MIRANDA

ABSTRACT. We extend de Finetti's (1937) notion of exchangeability to finite and countable sequences of variables, when a subject's beliefs about them are modelled using coherent lower previsions rather than (linear) previsions. We prove representation theorems in both the finite and the countable case, in terms of sampling without and with replacement, respectively. We also establish a convergence result for sample means of exchangeable sequences. Finally, we study and solve the problem of exchangeable natural extension: how to find the most conservative (point-wise smallest) coherent and exchangeable lower prevision that dominates a given lower prevision.

1. INTRODUCTION

This paper deals with belief models for both finite and countable sequences of exchangeable random variables taking a finite number of values. When such sequences of random variables are assumed to be exchangeable, this more or less means that the specific order in which they are observed is deemed irrelevant.

The first detailed study of exchangeability was made by de Finetti (1937) (with the terminology of 'equivalent' events). He proved the now famous Representation Theorem, which is often interpreted as stating that a sequence of random variables is exchangeable if it is conditionally independent and identically distributed (IID).[1] Other important work on exchangeability was done by, amongst many others, Hewitt and Savage (1955), Heath and Sudderth (1976), Diaconis and Freedman (1980) and, in the context of the behavioural theory of imprecise probabilities that we are going to consider here, by Walley (1991). We refer to Kallenberg (2002, 2005) for modern, measure-theoretic discussions of exchangeability.

One of the reasons why exchangeability is deemed important, especially by Bayesians, is that, by virtue of de Finetti's Representation Theorem, an exchangeable model can be seen as a convex mixture of multinomial models. This has given some ground (de Finetti, 1937, 1975; Dawid, 1985) to the claim that aleatory probabilities and IID processes can be eliminated from statistics, and that we can restrict ourselves to considering exchangeable sequences instead.[2]

De Finetti presented his study of exchangeability in terms of the behavioural notion of previsions, or fair prices. The central assumption underlying his approach is that a subject should be able to specify a fair price $P(f)$ for any risky transaction (which we shall call a *gamble*) f (de Finetti, 1974, Chapter 3). This is tantamount to requiring that he should always be willing and able to decide, for any real number r, between selling the gamble f for r, or buying it for that price. This may not always be realistic, and for this

Key words and phrases. Exchangeability, lower prevision, Representation Theorem, Bernstein polynomials, convergence in distribution, exchangeable natural extension, sampling without replacement, multinomial sampling, imprecise probability, coherence.

[1]See de Finetti (1975, Section 11.4); and Cifarelli and Regazzini (1996) for an overview of de Finetti's work.
[2]For a critical discussion of this claim, see Walley (1991, Section 9.5.6).

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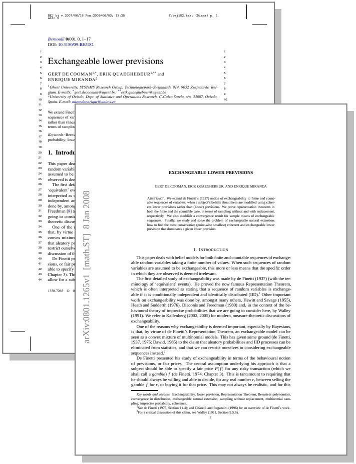
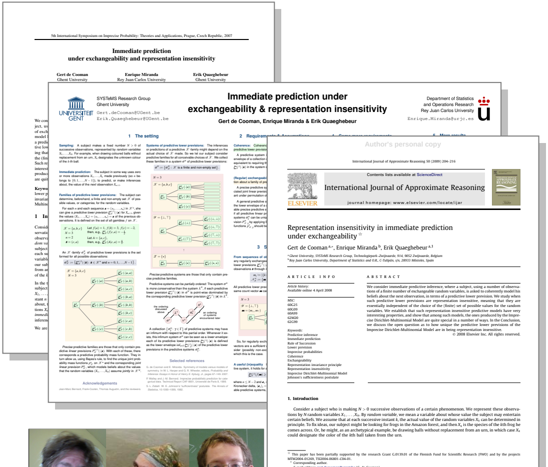
Exchangeable
lower previsions

- (in)finite sequences of finite-valued random variables
- exchangeability assessment & sequence order permutations
- sample sequences, count vectors & frequency vectors
- representation theorems
- exchangeable natural extension



Exchangeable lower previsions

Immediate prediction under exchangeability . . .


5th International Symposium on Imprecise Probability: Theories and Applications, Prague, Czech Republic, 2007

Immediate prediction under exchangeability and representation insensitivity

Gert de Cooman, Ghent University; Enrique Miranda, Rey Juan Carlos University; Erik Quaeghebeur, Ghent University

SYSTeMS Research Group, Ghent University — Gert.deCooman@UGent.be, Erik.Quaeghebeur@UGent.be

Immediate prediction under exchangeability & representation insensitivity

Gert de Cooman, Enrique Miranda & Erik Quaeghebeur

Department of Statistics and Operations Research, Rey Juan Carlos University — Enrique.Miranda@urjc.es

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Representation insensitivity in immediate prediction under exchangeability ☆

Gert de Cooman [a,*], Enrique Miranda [b], Erik Quaeghebeur [a,1]

[a] Ghent University, SYSTeMS Research Group, Technologiepark–Zwijnaarde, 914, 9052 Zwijnaarde, Belgium
[b] Rey Juan Carlos University, Department of Statistics and O.R., C-Tulipán, s/n, 28933 Móstoles, Spain

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Keywords: Predictive inference, Immediate prediction, Rule of Succession, Lower prevision, Imprecise probabilities, Coherence, Exchangeability, Representation invariance principle, Representation insensitivity, Imprecise Dirichlet-Multinomial Model, Johnson's sufficientness postulate

ABSTRACT

We consider immediate predictive inference, where a subject, using a number of observations of a finite number of exchangeable random variables, is asked to coherently model his beliefs about the next observation, in terms of a predictive lower prevision. We study when such predictive lower previsions are representation insensitive, meaning that they are essentially independent of the choice of the (finite) set of possible values for the random variables. We establish that such representation insensitive predictive models have very interesting properties, and show that among such models, the ones produced by the Imprecise Dirichlet-Multinomial Model are quite special in a number of ways. In the Conclusion, we discuss the open question as to how unique the predictive lower previsions of the Imprecise Dirichlet-Multinomial Model are in being representation insensitive.
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1. Introduction

Consider a subject who is making $N > 0$ successive observations of a certain phenomenon. We represent these observations by N random variables $X_1, \dots, X_N$. By *random variable*, we mean a variable about whose value the subject may entertain certain beliefs. We assume that at each successive instant k, the actual value of the random variables X_k can be determined in principle. To fix ideas, our subject might be looking for frogs in the Amazon forest, and then X_k is the species of the kth frog he comes across. Or, he might, as an archetypical example, be drawing balls without replacement from an urn, in which case X_k could designate the color of the kth ball taken from the urn.

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* Corresponding author. E-mail address: gert.decooman@ugent.be (G. de Cooman).
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Immediate prediction under exchangeability . . .


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Immediate prediction under exchangeability & representation insensitivity

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SYSTeMS Research Group, Ghent University — Gert.deCooman@UGent.be, Erik.Quaeghebeur@UGent.be

Department of Statistics and Operations Research, Rey Juan Carlos University — Enrique.Miranda@urjc.es

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[a] Ghent University, SYSTeMS Research Group, Technologiepark–Zwijnaarde, 914, 9052 Zwijnaarde, Belgium
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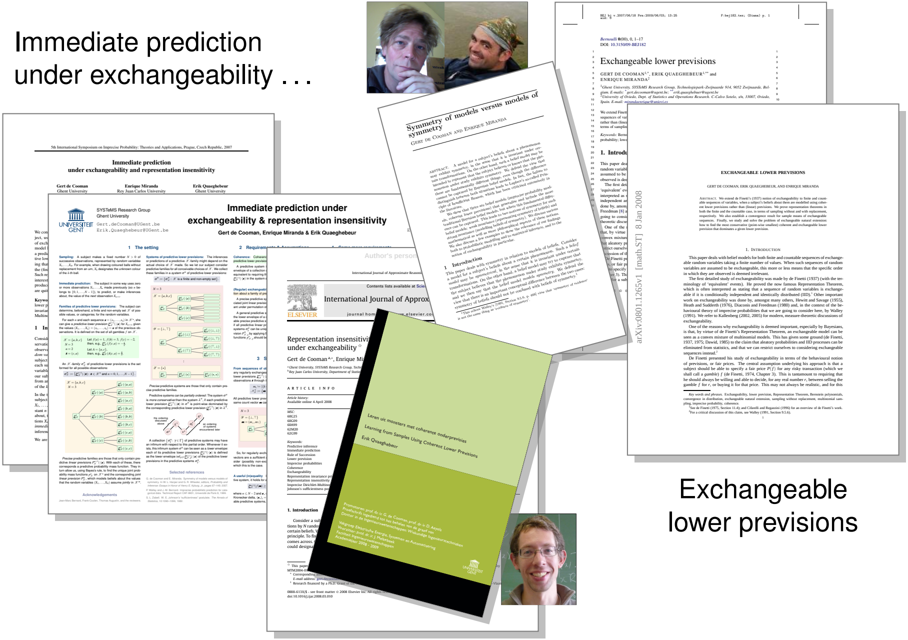
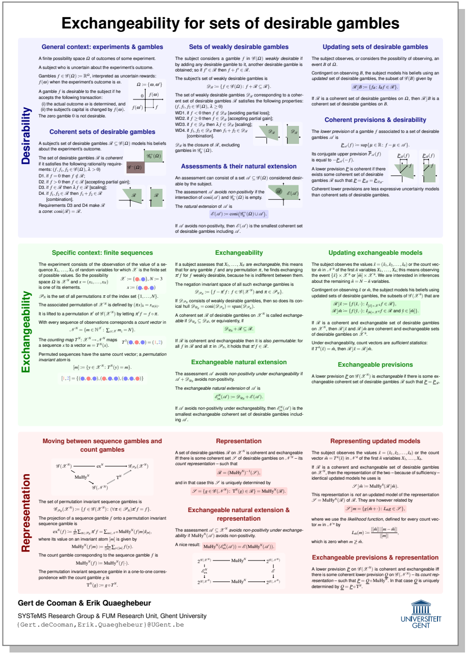
- families of exchangeable lower previsions
- coherent updated exchangeable lower previsions
- count vectors as sufficient statistics

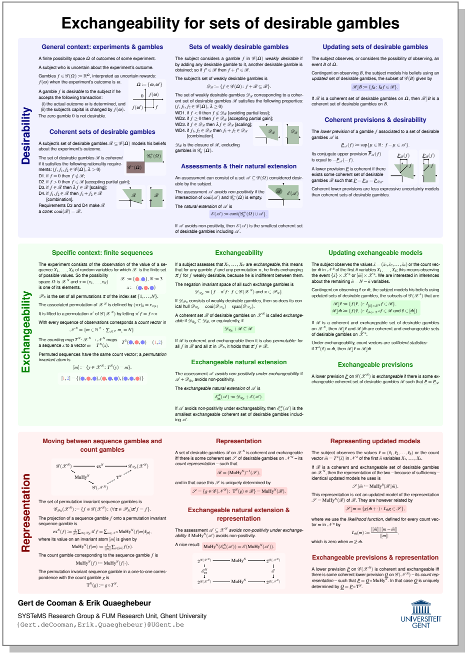
Immediate prediction under exchangeability . . .



Exchangeable lower previsions



- coherent sets of (weakly) desirable gambles
- natural extension
- updating
- relationship with previsions



Exchangeability for sets of desirable gambles

Desirability

General context: experiments & gambles

Coherent sets of desirable gambles

Sets of weakly desirable gambles

Assessments & their natural extension

Updating sets of desirable gambles

Coherent previsions & desirability

Exchangeability

Specific context: finite sequences

Exchangeability

Exchangeable natural extension

Updating exchangeable models

Exchangeable previsions

Representation

Moving between sequence gambles and count gambles

Representation

Exchangeable natural extension & representation

Representing updated models

Exchangeable previsions & representation

Gert de Cooman & Erik Quaeghebeur

SYSTeMS Research Group & FUM Research Unit, Ghent University

{Gert.deCooman,Erik.Quaeghebeur}@UGent.be

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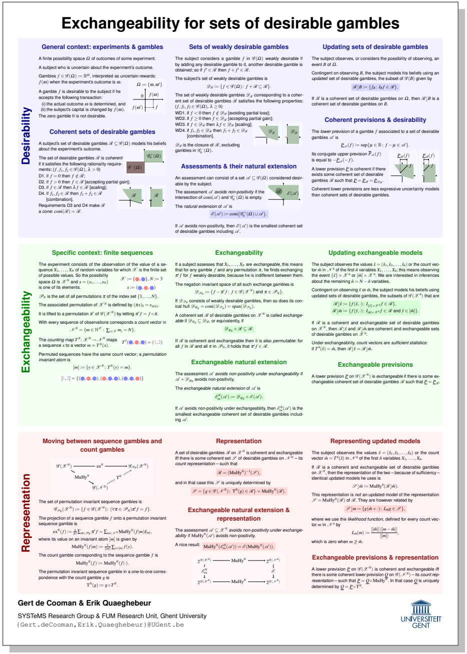
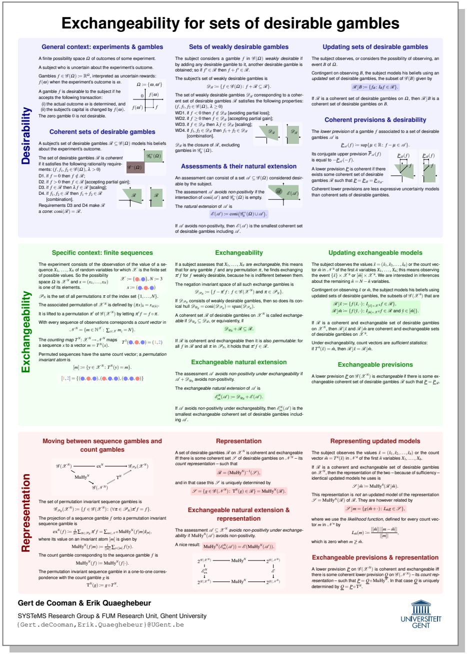
- exchangeability assessment & weakly desirable gambles
- exchangeable natural extension
- updating exchangeable models
- relationship with exchangeable previsions



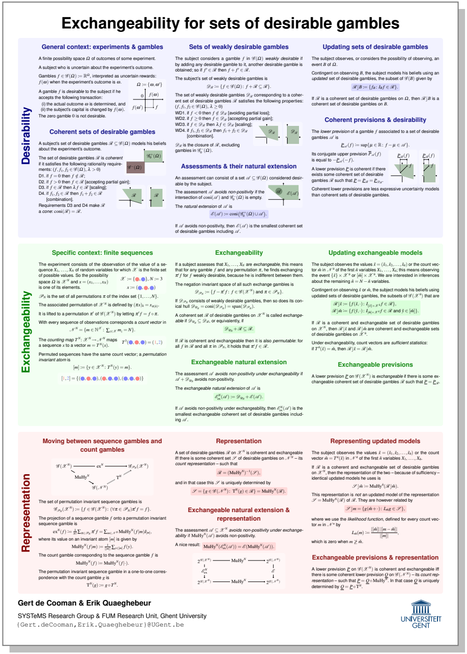
- representation theorem
- exchangeable natural extension & representation
- representing updated exchangeable models
- relationship with representing previsions



Best definition of coherence?

Infinite exchangeable sequences?

Representation for
infinite exchangeable sequences?



See you at the poster!