Noise quantization via possibilistic filtering

Kevin Loquin and Olivier Strauss

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Current position

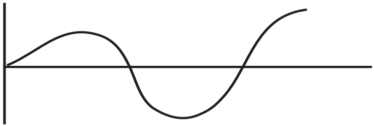
- Postdoc position at IRIT (Institut de Recherche en Informatique de Toulouse)
- In the team of Didier Dubois
- Geostatistics and Uncertainty

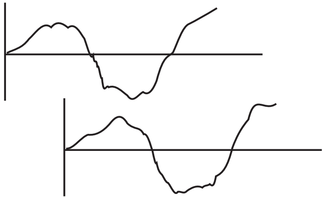
Position during the paper's writing

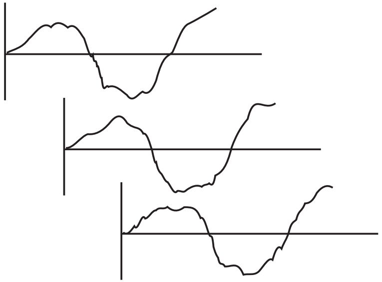
- PhD position at LIRMM (Laboratoire d'Informatique, de Robotique et de Microéléctronique de Montpellier)
- In the team of Olivier Strauss
- Signal processing and Uncertainty
- Organizer of SIPTA school 08 in Montpellier

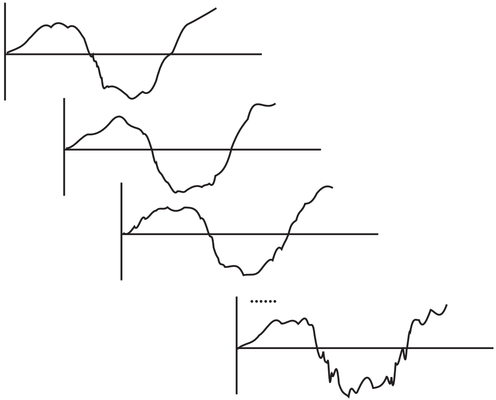
Permanent position

- Lecturer at LIRMM (Laboratoire d'Informatique, de Robotique et de Microéléctronique de Montpellier)
- Signal and Image Processing and Uncertainty
- Organizer of SIPTA school 08 in Montpellier









1. **Noise :** Statistical variations of the signal

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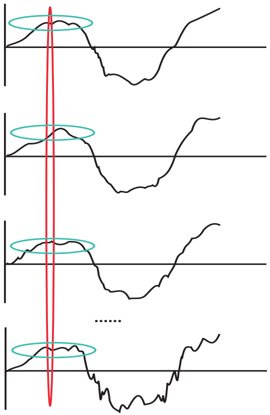
2. **Noise quantifiers :** statistical variance or standard deviation
3. **A common problem :** how to quantify the noise on a unique signal aquisition ?



The local spatial or temporal variations, i.e. the variations inside a neighborhood, are equal to the statistical variations, i.e. to the noise.

$\Longrightarrow$ usual noise level quantifiers are the local variance or standard deviation

Definition

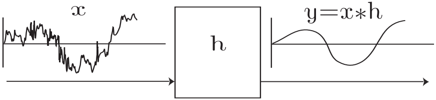
x : input signal

h : impulse response of the filter

y : output (or filtered) signal

$$y_k = \sum_{i \in \mathbb{Z}} x_i h_{k-i} = (x * h)_k = \sum_{i \in \mathbb{Z}} x_i h_i^k.$$

Notation : $h_\bullet^k = h_{k-\bullet}$.



Aim : It consists in selecting the low frequency part from the input signal.

How to : A smoothing (or low-pass) filter sets each location *k* to the average value, or a weighted average, of itself and its nearby neighbors.

The considered filters, whose impulse responses are denoted by κ are generally

- $\kappa^k \geq 0$,
- $\sum_{i \in \mathbb{Z}} \kappa_i^k = 1$.

They are called smoothing convolution kernels and can be seen as **probability distributions**.

Remark : they are generally symmetric.

A probability measure can thus be associated to a smoothing convolution kernel :

$$\forall A \subseteq \mathbb{Z},\ P_{\kappa^k}(A) = \sum_{i \in A} \kappa_i^k .$$

The filtering of a signal can thus be written as an expectation operator :

$$y_k = \sum_{i \in \mathbb{Z}} x_i \kappa_i^k = \mathbb{E}_{\kappa^k}(x).$$

The kernels are **possibilistic**

$$\sup_{i\in\mathbb{Z}} \pi_i^k = 1.$$

Possibility measure (concave capacity) :

$$\forall A \subseteq \mathbb{Z},\ \Pi_{\pi^k}(A) = \sup_{i\in A} \pi_i^k,$$

A possibilistic kernel π^k can encode a special family of smoothing convolution kernels, denoted by $\mathcal{M}(\pi^k)$ and defined by

$$\mathcal{M}(\pi^k) = \left\{\kappa^k \mid \forall A \subseteq \mathbb{Z},\ N_{\pi^k}(A) \leq P_{\kappa^k}(A) \leq \Pi_{\pi^k}(A)\right\}.$$

Definition

the imprecise output of possibilistic low-pass filter is given by :

$$[\underline{y}_k, \overline{y}_k] = [\mathbb{C}^c_{\pi^k}(x), \mathbb{C}_{\pi^k}(x)].$$

where

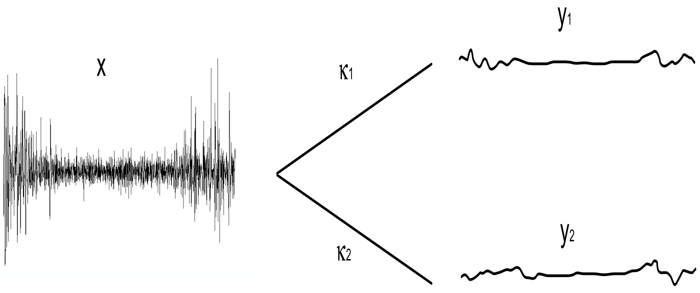
$$\mathbb{C}^c_{\pi^x}(x) = (C)\int_\Omega x dN_\pi,$$
$$\mathbb{C}_{\pi^x}(x) = (C)\int_\Omega x d\Pi_\pi.$$

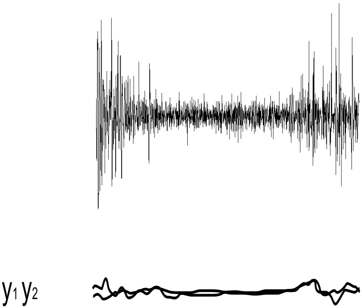
Theorem

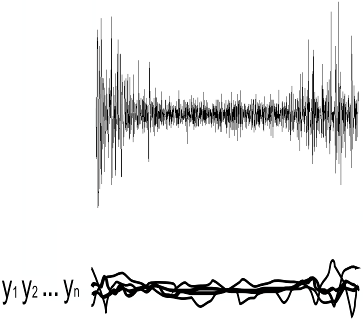
(Schmeidler and Denneberg)
If π^k is a possibilistic kernel on $\mathbb{Z}$ then $\forall \kappa^k \in \mathcal{M}(\pi^k)$,

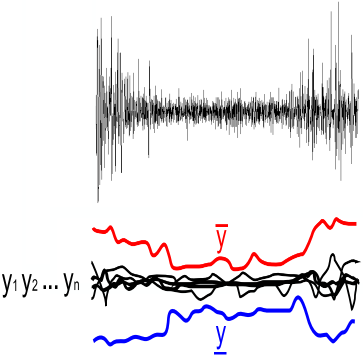
$$y_k \in [\underline{y}_k, \overline{y}_k].$$

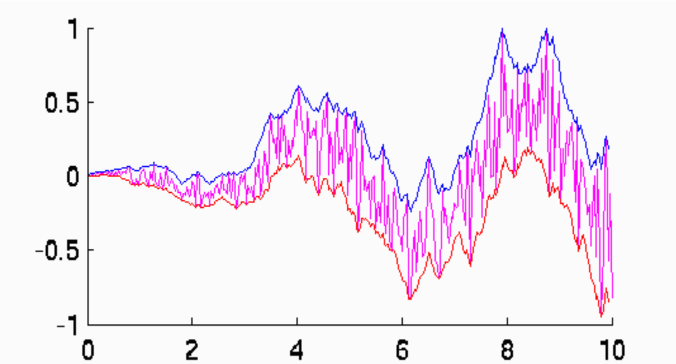
Besides, the bounds are reached.


X
κ1
κ2
y1
y2


y1 y2


y1 y2 ... yn


$\overline{y}$
$y_1\, y_2 \ldots y_n$
$\underline{y}$


1
0.5
0
-0.5
-1
0
2
4
6
8
10

Definition

Under the local ergodicity assumption, we propose to estimate the noise level on the input :

$$\lambda_k = \overline{y}_k - \underline{y}_k. \tag{1}$$

- Experiments with real and simulated noise.
- Comparison of our approach with local standard deviation estimates.
- The results of an algorithm of edge detection of an image based on possibilistic filtering are presented.
- Our method is compared with usual Canny-Deriche filtering approaches in presence of noise.

- A new filtering approach that handles epistemic uncertainty on the filter to use.
- In image processing, we proved equivalence between this approach and a particular morphological operator (lower bound $\Leftrightarrow$ erosion and upper bound $\Leftrightarrow$ dilation).
- Also applied in signal sampling and interpolation, rigid image motion, edge detection, measurement modelling.
- Methods that handle the noise in more tractable way than usual.