Aggregating Imprecise Probabilistic Knowledge

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ISIPTA '09 - Durham

July 17th, 2009

About us

Alessio (Benavoli)

- Control and estimation
- Belief functions/random sets
- Tracking and filtering

Alessandro (Antonucci)

- Uncertain reasoning
- Credal sets/nets
- Risk analysis

Motivations for this paper

- Information fusion by coherent lower previsions (CLPs)
- Our first attempt to "play" with CLPs
  (and a pedagogical example of how this can be done)

Aggregation

- Two (or more) sources providing information about X
- Information aggregated by the fusion center (FC)
- Bayesian case (probabilities multiply)
- Belief functions (masses multiply)
- ...
- Credal sets (next talk)
- Coherent lower previsions (this talk)

$\underline{P}_1^X$ $P_1(X)$ $m_1(x)$ Source A_1 → Fusion Center

$\underline{P}_2^X$ $P_2(X)$ $m_2(x)$ Source A_2 → Fusion Center

$\underline{P}_0^X = F[\underline{P}_1^X, \underline{P}_2^X]$

$P_0(x) \propto P_1(x) \cdot P_2(x)$

$m_0(x) \propto \sum_{x_1 \cap x_2 = x} m_1(x_1) \cdot m_2(x_2)$

Bayesian combination

- An auxiliary variable for each source
- Sources return conditionals
- A prior for each source
- This defines a joint for each source
- Conditionals through Bayes' rule
- Model revision of FC from sources
- Sources independent given X
- FC has its own prior
- FC joint by chain rule
  $P_0(X)P_0(A_1|X)P_0(A_2|X)$
- Aggregated posterior by Bayes'
  $P_0(X|a_1, a_2) \propto P_0(X)P_0(X|a_1)P_0(X|a_2)$

sources

A_1	A_2
$P_1(X\|a_1)$	$P_2(X\|a_2)$
$P_1(A_1)$	$P_2(A_2)$
↓	↓
$P_1(X, A_1)$	$P_2(X, A_2)$
↓	↓
$P_1(A_1\|X)$	$P_2(A_2\|X)$
↓	↓
FC $P_0(A_1\|X)$	$P_0(A_2\|X)$

↘ ↙

$P_0(A_1, A_2|X)$ $\quad P_0(X)$

↓ ↙

$P_0(X, A_1, A_2)$

↓

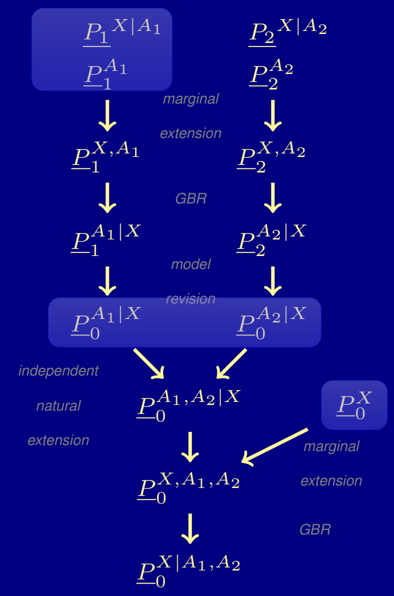
$P_0(X|a_1, a_2)$

Extension to coherent lower previsions

- Each source returns a conditional CLP ($\underline{P}_1^{X|A_1}$ and $\underline{P}_2^{X|A_2}$)
- Sources priors are also CLPs ($\underline{P}_1^{A_1}$ and $\underline{P}_2^{A_2}$)
- Joint by marginal extension $\underline{P}_j^{X,A_j}(f_j) := \underline{P}_j^{A_j}\left(\underline{P}_j^{X|A_j}(f_j|A_j)\right)$
- Conditionals $\underline{P}_1^{A_1|X}$ and $\underline{P}_2^{A_2|X}$ by GBR/regular extension
- Model revision from sources to FC $\underline{P}_0^{A_j|X} \equiv \underline{P}_j^{A_j|X}$
- Epistemic irrelevance of the sources given X, conditional $\underline{P}_0^{A_1,A_2|X}$ by independent natural extension
- FC has its own prior CLP $\underline{P}_0^X$
- Joint by marginal extension $\underline{P}_0^{X,A_1,A_2}(g) := \underline{P}_0^X\left(\underline{P}_0^{A_1,A_2|X}(g|X)\right)$
- Aggregated posterior CLP $\underline{P}_0^{X|A_1,A_2}$ by GBR

Checking coherence

- Coherence required only "separately" for the sources and the FC
- Sources coherence: trivial because of marginal extension
- FC coherence: *the separately coherent conditional lower previsions $\underline{P}_0^{A_j|X}$ and $\underline{P}_0^X$ are jointly coherent* (proof in the paper)



A closed formula for linear vacuous mixtures

- Explicit computation for a special class of CLPs
  $\underline{P}_j^{X|A_j}(f_j|a_j) := \epsilon_j^{a_j} \sum_{x\in\mathcal{X}} p_j(x|a_j) f_j(x,a_j) + (1-\epsilon_j^{a_j}) \min_{x\in\mathcal{X}} f_j(x,a_j)$
- If the FC has a vacuous prior, it will never learn from the sources
  ($\underline{P}_0^X$ is vacuous $\Rightarrow \underline{P}_0^{X|A_1,A_2}$ vacuous)
- If the sources are vacuous, the FC keeps its prior as a posterior
  ($\underline{P}_1^{X|A_1}$ and $\underline{P}_2^{X|A_2}$ vacuous $\Rightarrow \underline{P}_0^{X|A_1,A_2} = \underline{P}_0^X$)
- In general $P_0^{X|A_1,\ldots,A_n}(g|a_1,\ldots,a_n)$ is the solution μ of

$$0 = \epsilon_0 \sum_{x\in\mathcal{X}} \Big\{\Big[\underline{P}_0^{A_1|X}(I_{\{\tilde{a}_1\}}|x)\cdots \underline{P}_0^{A_n|X}(I_{\{\tilde{a}_n\}}|x) I_{\{g(x,\tilde{a}_1,\ldots,\tilde{a}_n)-\mu\geq 0\}}$$
$$+\overline{P}_0^{A_1|X}(I_{\{\tilde{a}_1\}}|x)\cdots \overline{P}_0^{A_n|X}(I_{\{\tilde{a}_n\}}|x) I_{\{g(x,\tilde{a}_1,\ldots,\tilde{a}_n)-\mu< 0\}}\Big](g(x,\tilde{a}_1,\ldots,\tilde{a}_n)-\mu)p_0(x)\Big\}$$
$$+(1-\epsilon_0)\min_{x\in\mathcal{X}} \Big\{\Big[\underline{P}_0^{A_1|X}(I_{\{\tilde{a}_1\}}|x)\cdots \underline{P}_0^{A_n|X}(I_{\{\tilde{a}_n\}}|x) I_{\{g(x,\tilde{a}_1,\ldots,\tilde{a}_n)-\mu\geq 0\}}$$
$$+\overline{P}_0^{A_1|X}(I_{\{\tilde{a}_1\}}|x)\cdots \overline{P}_0^{A_n|X}(I_{\{\tilde{a}_n\}}|x) I_{\{g(x,\tilde{a}_1,\ldots,\tilde{a}_n)-\mu< 0\}}\Big](g(x,\tilde{a}_1,\ldots,\tilde{a}_n)-\mu)\Big\}$$

Application to Zadeh's paradox

- A patient disease X (meningitis, concussion or brain tumor)
- Two doctors (sources) A_1 and A_2
- Doc A_1: 99% meningitis, 1% brain tumor, concussion cannot be
- Doc A_2: 99% concussion, 1% brain tumor, meningitis cannot be
- After aggregation, either Dempster' and Bayesian combinations say tumor 100%
- Our aggregated diagnosis is $\underline{P}_0^{X|A_1,A_2}$
- If both doctors are reliable, $\underline{P}_0(X|a_1,a_2)$ as in the Bayesian case
- Conflict: (at least) one of them should be unreliable
- We compute $\underline{P}_0(X|\{\neg a_1, a_2\} \cup \{a_1, \neg a_2\} \cup \{\neg a_1, \neg a_2\})$
- The patient suffers from either concussion or meningitis!