On the use of a new discrepancy measure to correct incoherent assessments and to aggregate conflicting opinions based on imprecise conditional probabilities

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About Giuliana..

- Position: Associate Professor in Probability & Statistics
- Research interests:
- Robust Inference in Probability under Vague Information
- Uncertainty qualitative orderings and their representation
- Bivariate exponential distributions
- Asymptotic distribution of density ratios
- Discrepancy measures among conditional probabilities

About Francesca..

- Position: PhD student in Mathematics and Computer Science
- Research interests:  ?

About me..

- Position: Researcher and “aggregate” Professor in Probability & Statistics
-  Research interests:

“partial” models

- Inference via coherent conditional <u>assessments</u>
- Uncertainty qualitative orderings
- Use of logical tools in coherent probability models to reduce computational complexity
- Correction of incoherent conditional assessments

About the origins of the paper…

Coherent correction of inconsistent conditional probability assessments

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Abstract

In this paper we suggest a procedure to adjust an incoherent conditional probability assessment given on a partial domain. We look for a solution that tries to attain two separate goals: on one hand the solution should be as close as possible to the initial assessments, on the other hand we do not want to insert more information than we had at the beginning. The first goal is achieved by minimizing an appropriately defined distance among assessments, while for the second we look for a "maximum entropy" like solution.

Keywords: Conditional probability coherence, Divergence, Scoring rules.

1 Introduction

In practical applications it is natural to give evaluations of probability only of relevant events; moreover it can happen that these evaluations do not fit well with each other, especially when coming from different sources. Another common feature is that events are judged under specific circumstances, implying a conditional assessment. Often the assessment is intended to be used for inference purposes, i.e. to see how a further (conditional) event can be evaluated consistently with the initial assessment. Of course, the inferential results are meaningful only if the prior information encompassed in the initial assessment is coherent by itself. If not, a modification is required. Usually such a problem is solved with a revision of the initial evaluations. We propose a methodology for choosing an assessment correction automatically. A similar proposal can be found in Kriz [9].

Therefore our input consists of an incoherent conditional probability assessment given on a partial domain. We want to find a coherent assessment on the same domain that will preserve the opinion expressed by the initial assessment as much as possible, without introducing exogenous information. This goal is obtained by minimizing some kind of distance among partial conditional assessments.

(Pseudo)distances among probability distributions are usually measured through divergencies (e.g. Euclidean distance, Kulback-Leibler divergence, Csiszár f-divergences, etc.). Some of them can be applied only among unconditional full probability distributions; others could be applied to our context of partial conditional assessments (see for example [9]), but do not have any probabilistic justification, being purely geometrical tools. Hence, for our purpose, in this paper we introduce an index of "discrepancy" among partial conditional probability assessments which is derived by a particular scoring rule. Such a scoring rule is inspired by the one, introduced by Lad in [11] for unconditional probability distributions, and adapted here to conditional-logical arguments.

Independently of the divergence used to extrapolate the closest coherent assessment, for inference purposes, among all the compatible

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Correction of incoherent conditional probability assessments

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1 Introduction

In this paper we deal with incoherent partial conditional probabilities assessments. Such kind of evaluations arise because often it is natural to give evaluations of probability only of relevant events, that are judged under specific circumstances. And it can happen that the numerical values do not fit well with each other, especially when information comes from different sources.

Inconsistency, if not adjusted, can be dangerous. In fact, often the assessment is intended to be used for inference purposes, i.e. to see how a further (conditional) event can be evaluated consistently with the initial assessment. Of course, the inferential results are meaningful only if the prior information encompassed in the initial assessment is coherent by itself.

Hence it is quite natural to search for a coherent assessment on the same domain that will preserve the opinion expressed by the initial assessment as much as possible, without introducing exogenous information. This goal is obtained by minimizing some kind of distance among partial conditional assessments.

Distances and pseudo-distances among probability distributions are usually measured through divergencies (e.g. Euclidean dis-

About the paper…

- **Framework:** conditional probability assessments
  - domain $\mathcal{E} = [E_1|H_1, \ldots, E_n|H_n]$
  - interval probabilities $\mathbf{lub} = ([lb_1, ub_1], \ldots, [lb_n, ub_n])$
  - logical dependencies (incompatibilities, implications, ...)
- **Aims:**

(incurring in a uniform loss)

1. To correct incoherent assessments

empty (credal) set of compatible full conditional distributions

$$\mathcal{M} \quad := \quad \{P \text{ coherent} \,|lb_i \leq P(E_i|H_i) \leq ub_i, i = 1, \ldots, n\} = \emptyset$$

2. To aggregate conflicting opinions

different coherent sources of information

$$(\mathcal{E}^s = [E_{1.s}|H_{1.s}, \ldots, E_{n.s}|H_{n.s}], \mathbf{lub}^s = ([lb_{1.s}, ub_{1.s}], \ldots, [lb_{n.s}, ub_{n.s}])) \quad s \in S$$

incoherent aggregation

$$\begin{cases} \mathcal{E} = \bigcup_{s \in S} \mathcal{E}^s \\ \\ \mathbf{lub} = \bigcup_{s \in S} \mathbf{lub}^s \end{cases} \quad + \quad \text{some coincidence relations} \begin{cases} E_{i.s_j} H_{i.s_j} \equiv E_{i.s_k} H_{i.s_k} \\ \\ H_{i.s_j} \equiv H_{i.s_k} \end{cases}$$

About the main tool…

- The discrepancy

$$q_i = \frac{\sum_{j:\,\omega_j \subset E_i H_i} \alpha_j}{\sum_{j:\,\omega_j \subset H_i} \alpha_j}$$

$$\Delta(\mathbf{p}, \boldsymbol{\alpha}) := \sum_{\alpha(H_i)>0} \alpha(H_i) \left(q_i \ln\left(\frac{q_i}{p_i}\right) + (1 - q_i) \ln\left(\frac{1-q_i}{1-p_i}\right) \right)$$

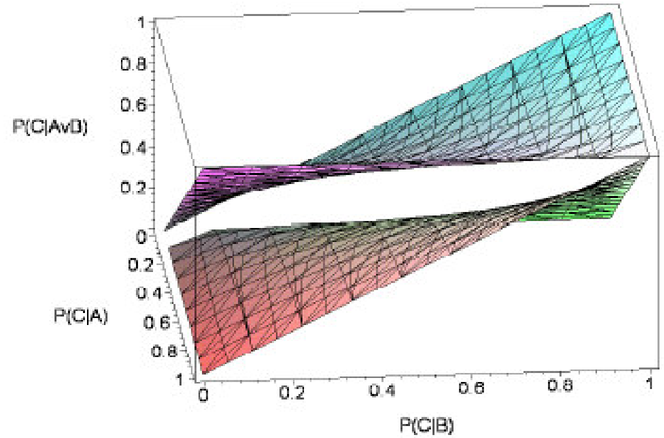
conditional assessment

unconditional prob. distr.

- Originated by the scoring rule

$$S(\mathbf{p}) := \sum_{i=1}^{n} |E_i H_i| \ln p_i + \sum_{i=1}^{n} |\neg E_i H_i| \ln(1 - p_i)$$

- Discrepancy vs Divergence
  - Coherent sets of conditional assessments (in general) not convex



About the procedure for imprecise cond. prob.…

- Iteration of parametric optimization problems $f \in \{1, \ldots, n\}$

$$\text{minimize } \Delta(\mathbf{v}, \boldsymbol{\alpha})$$ ⇨ **the discrepancy**

under the constraints

$$v_f = lb_f \quad \textbf{or} \quad v_f = ub_f$$ ⇨ **one bound fixed**

$$\forall i \neq f \quad lb_i \leq v_i \leq ub_i$$ ⇨ **others remain intervals**

$$\sum_{j:\omega_j \subset E_k H_k} \alpha_j = q_k \sum_{j:\omega_j \subset H_k} \alpha_j$$ ⇨ **coherence constraints**

$$\boldsymbol{\alpha} \in \mathcal{A}_0$$ ⇨ **normalization to $H^0 = \vee H_i$**

- obtaining $2n$ coherent precise assessments…

$$\mathcal{Q} = \{\underline{\mathbf{q}}_f, \overline{\mathbf{q}}_f, f = 1, \ldots, n\}$$

- …whose lower/upper envelope give the solution

$$lc_i := \min_{\tilde{\mathbf{q}} \in \mathcal{Q}} \tilde{\mathbf{q}}(E_i|H_i) \qquad uc_i := \max_{\tilde{\mathbf{q}} \in \mathcal{Q}} \tilde{\mathbf{q}}(E_i|H_i)$$

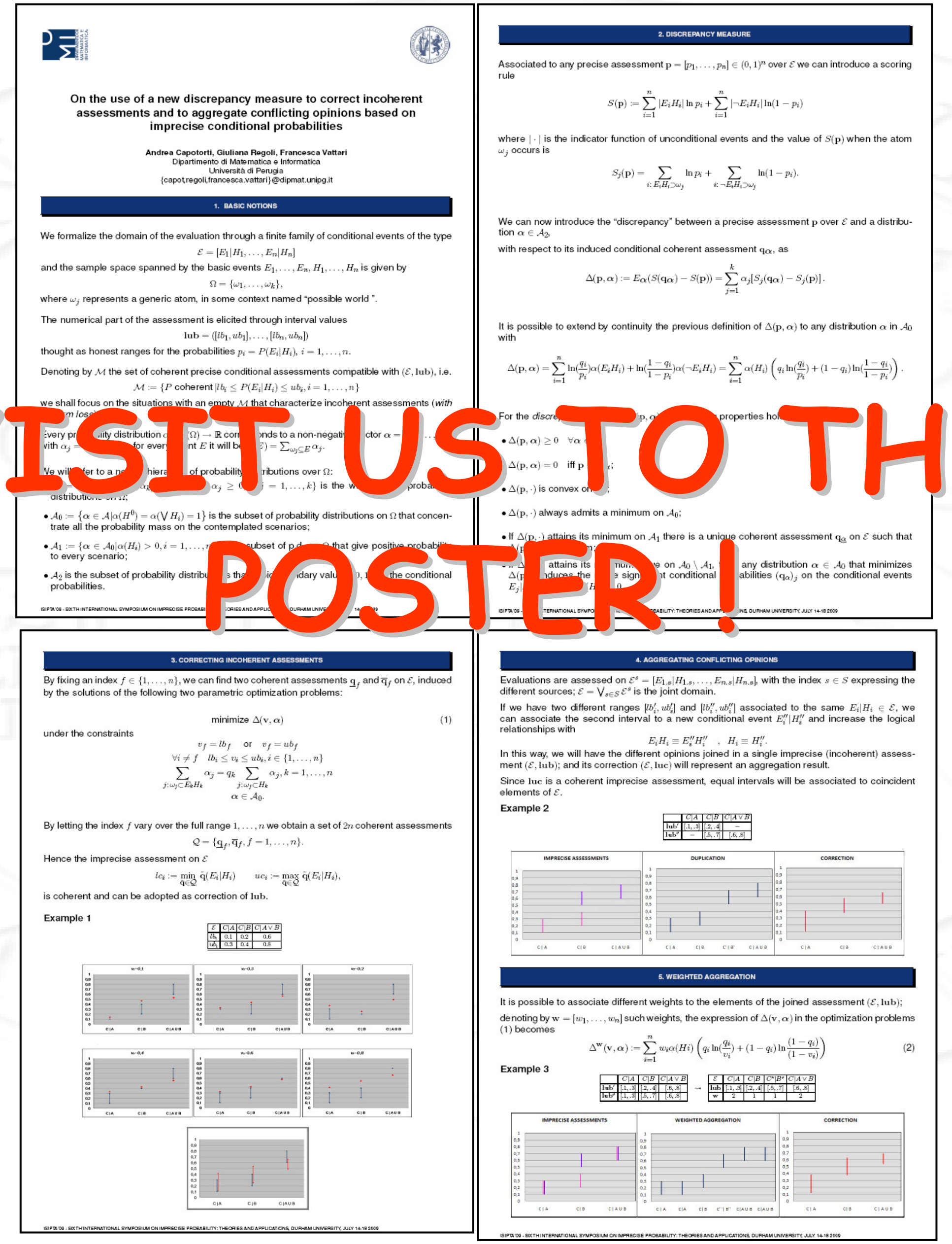
Final remarks

- Discrepancy can be generalized to a "weighted" version…. $\Delta^{\mathbf{w}}(\mathbf{v},\boldsymbol{\alpha}) := \sum_{i=1}^{n} w_i \alpha(Hi)\left(q_i \ln(\frac{q_i}{v_i}) + (1-q_i)\ln(\frac{(1-q_i)}{(1-v_i)}) \right)$
  - This is a preliminary study. In particular:
  - some theoretical details must be fixed

    (e.g. uniqueness of **luc**)
  - comparison with different aggregation operators is needed

    (especially with respect to practical applications)
  - for large scale problems, computational complexity must be overcome by heuristics

For details and simple examples….



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