

# Consistent approximations of BF & Credal semantics of Bayesian transformations

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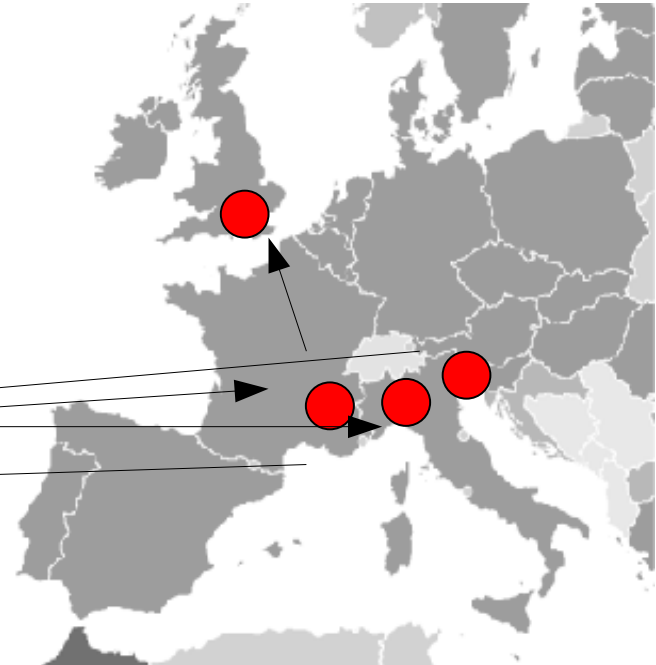
# About me

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- Master's thesis on gesture recognition at the **University of Padova**
- Visiting student, ESSRL, **Washington University in St. Louis**
- Ph.D. thesis on a geometric approach to belief functions
- Researcher at **Politecnico di Milano** with the Image and Sound Processing group
- Post-doc at the **University of California at Los Angeles**, UCLA Vision Lab
- Marie Curie fellow at **INRIA Rhone-Alpes**, Grenoble
- Lecturer, **Oxford Brookes University**, Oxford

# About me

- Graphical representation





# My background

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## Machine Learning

- learning Riemannian metrics

## Discrete Mathematics

- abstract independence

## research

## Computer Vision

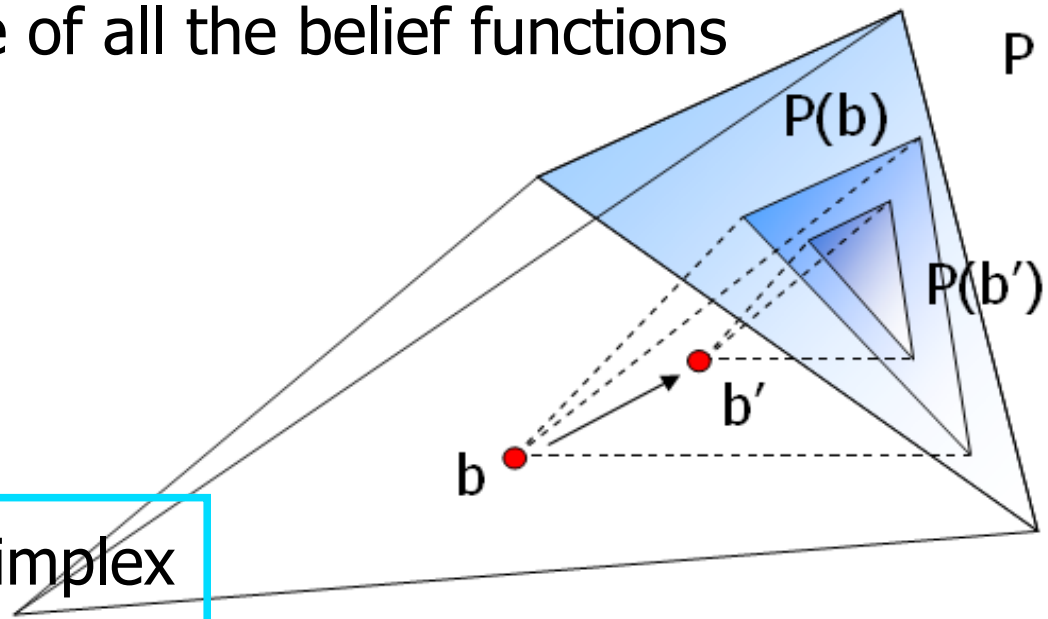
- motion analysis
  - gesture and action recognition
  - object pose estimation
  - gait identification
  - bodypart segmentation

## Belief functions and Imprecise Probabilities

- geometric approach to uncertainty theories
- approximation problem

# A geometric approach to uncertainty

- belief space: the space of all the belief functions on a given frame

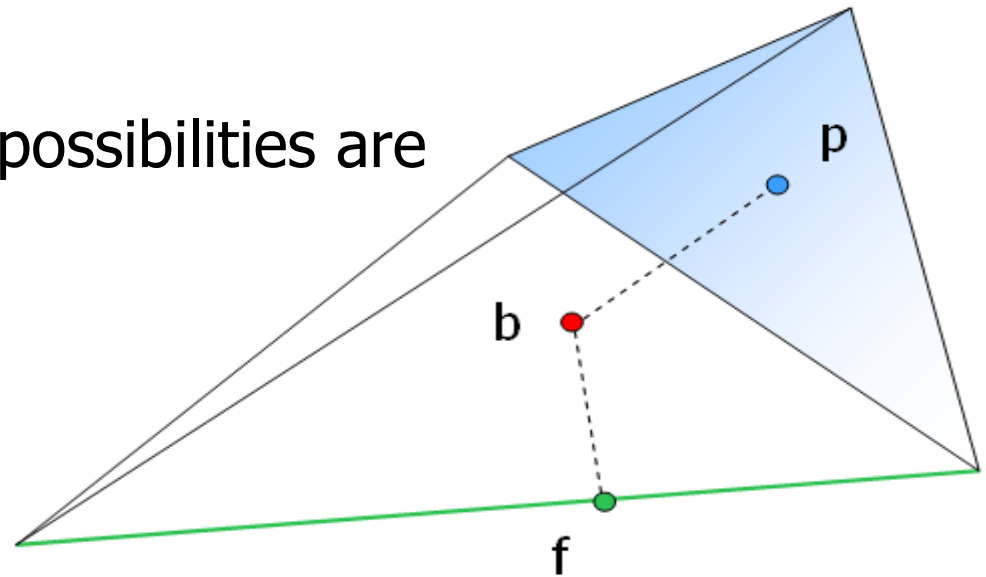


- it has the shape of a simplex
- IEEE Tr. SMC-C '08, Ann. Combinatorics '06, FSS '06, IDA'09

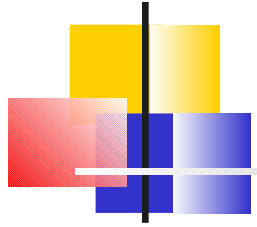
# Approximation problem

- how to transform a measure of a certain family into a different uncertainty measure → can be done geometrically

- Probabilities, fuzzy sets, possibilities are all special cases of b.f.s



- IEEE Tr. SMC-B '07, IEEE Tr. Fuzzy Systems '07, AMAI '08, AI '08, IEEE Tr. SMC-B '09



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# credal semantics of Bayesian transformations

# Two families of probability transformations (or three..)

- **Pignistic function** i.e. center of mass of consistent probabilities
- **orthogonal projection** of  $b$  onto  $P$
- **intersection probability**

- commute with Dempster's combination

- commute with affine combination

- **Relative plausibility** of singletons
- **Relative belief** of singletons [IEEE TFS08]
- Relative **uncertainty** of singletons [AMAI08]





# Three different credal sets

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- each transformation is indeed a transformation of an upper, lower, or interval probability system
- they have a credal interpretation

$$T^1[b] \doteq \{p : p(x) \geq b(x) \quad \forall x \in \Theta\}$$

→  $\tilde{b}$

$$T^{n-1}[b] \doteq \{p : p(x) \leq pl_b(x) \quad \forall x \in \Theta\}$$

→  $\tilde{pl}_b$

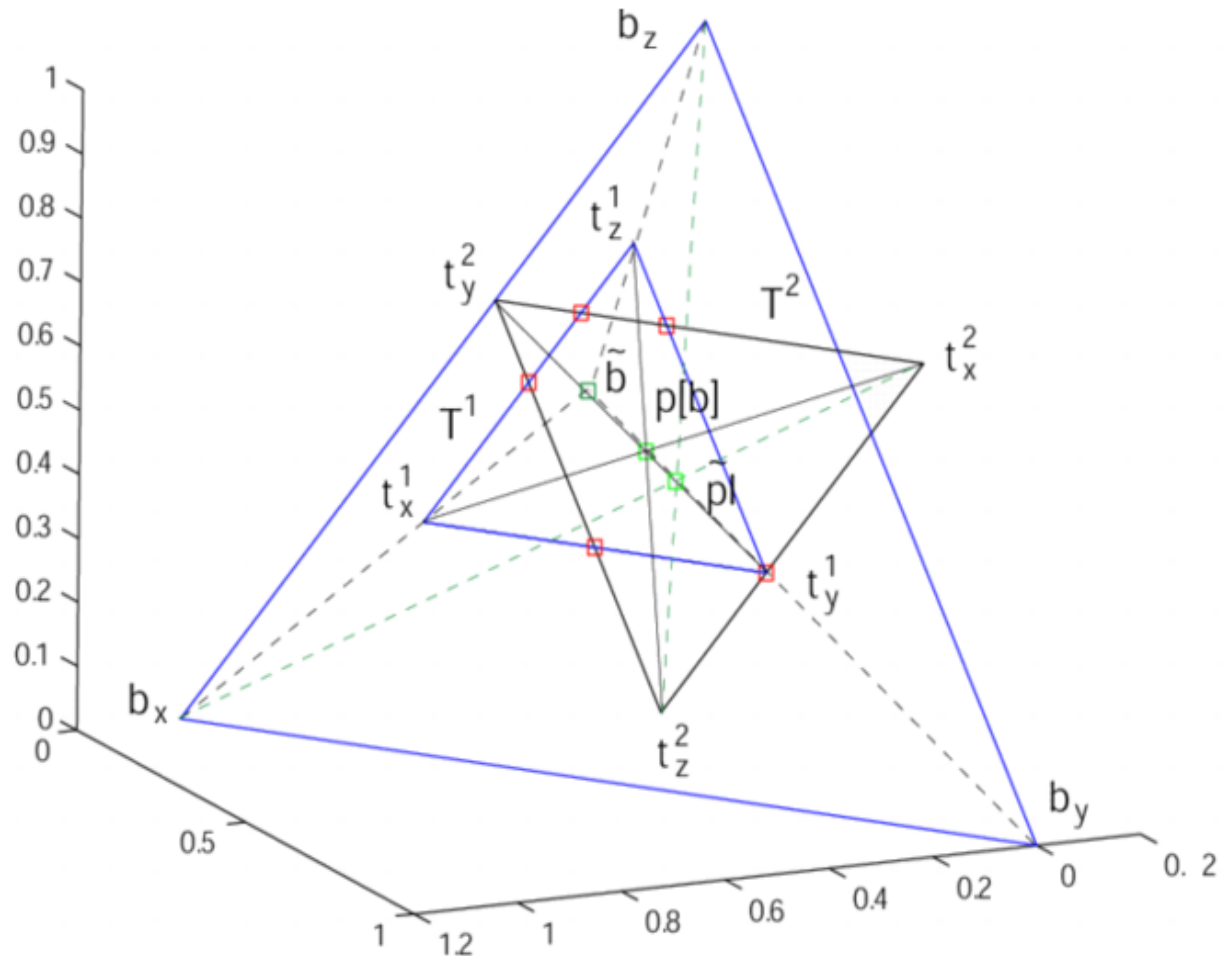
$$\mathcal{P}[b, pl_b] \doteq \{p \in \mathcal{P} : b(x) \leq p(x) \leq pl_b(x), \forall x \in \Theta\}.$$

→  $p[b]$

- interpretation of the associate transformations?

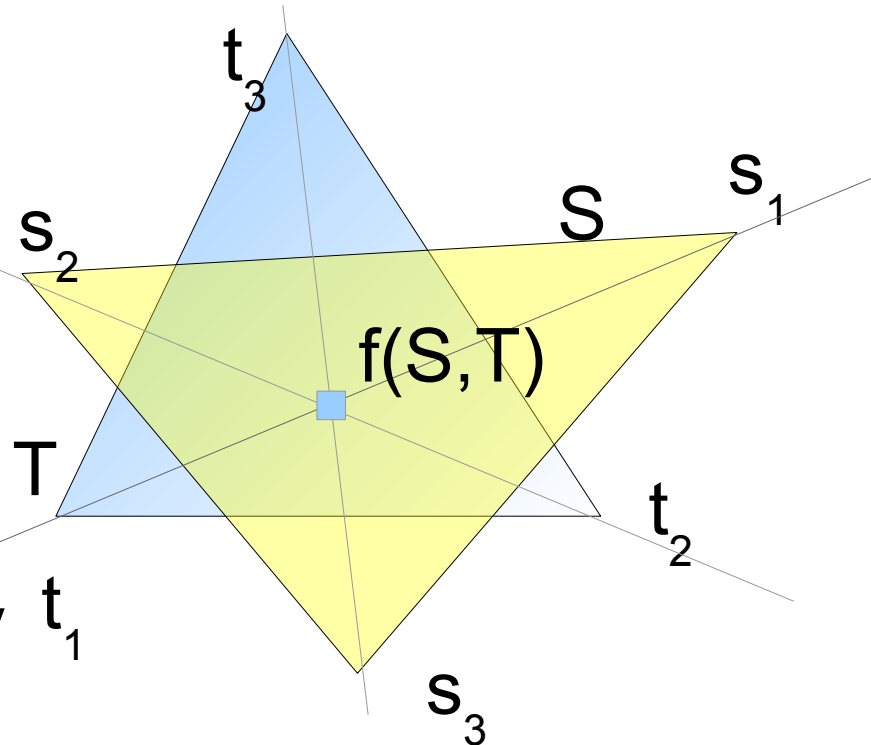
# Bayesian transformations as foci

- relative belief = focus of  $(P, T^1)$
- relative plausibility = focus of  $(P, T^{n-1})$
- intersection probability = focus of  $(T^1, T^{n-1})$



# Focus of a pair of simplices

- different Bayesian transformations can be seen as **foci** of a pair of simplices among  $(P, T^1, T^{n-1})$
- focus = point with the **same simplicial coordinates** in the two simplices
- rationality principle: only distribution that **meet both constraints in the same way**

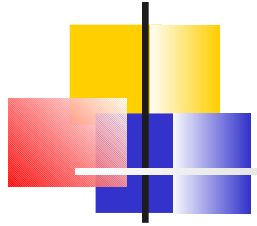




# TBM-like frameworks

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- **Transferable Belief Model:** belief are represented as credal sets, decisions made after pignistic transformation [Smets]
- reasoning frameworks similar to the TBM can be imagined ...
- ... in which upper, lower, and interval constraints are repr. as credal sets ...
- ... while decisions are made after appropriate transformation



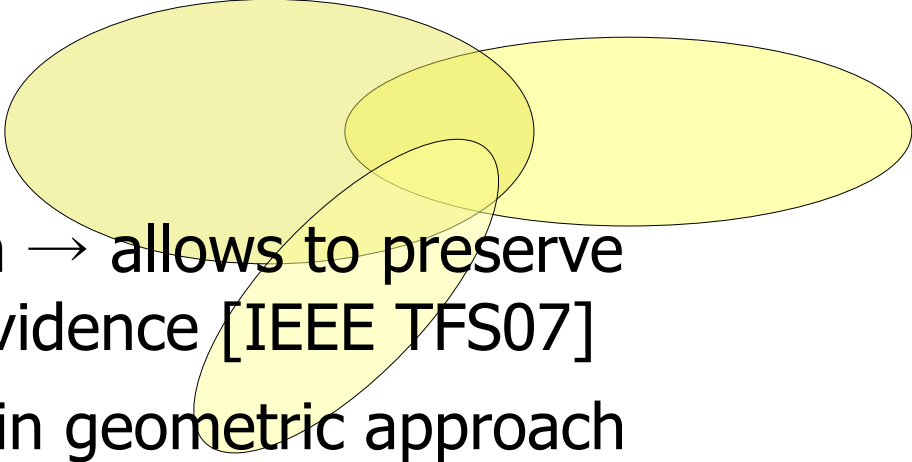
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consistent approximations  
of belief functions



# Consistent belief functions

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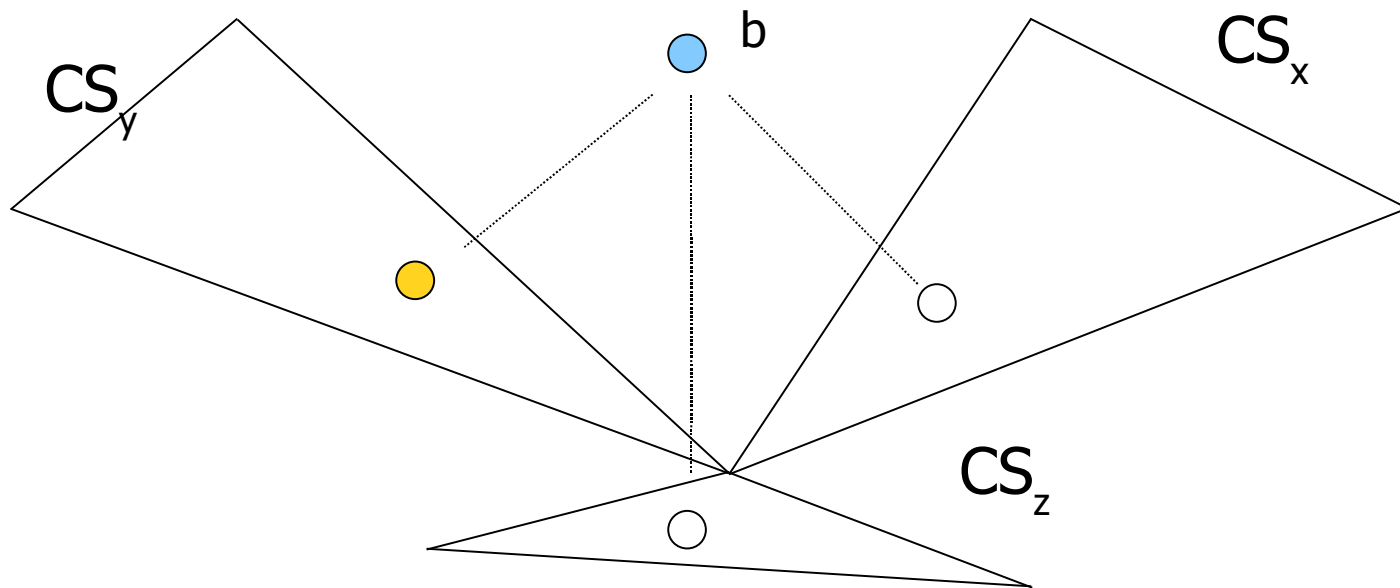
- Bfs are result of aggregation of **conflicting** pieces of evidence
  - **consistent bfs**  $\leftrightarrow$  **consistent knowledge bases**
    - (cannot derive incompatible conclusions from them)
  - BFs whose focal elements have non-empty intersection
  - **internal conflict is null**
  - **consistent approximation**  $\rightarrow$  allows to preserve consistency of the body of evidence [IEEE TFS07]
  - can be done using  $L_p$  norms in geometric approach
- 



# Projection onto a complex

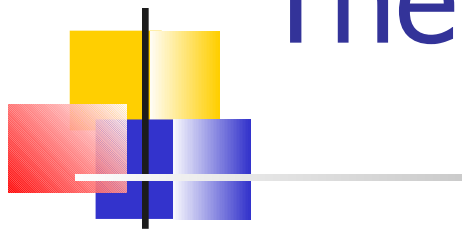
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- they live on a simplicial complex
- idea: belief function has a **partial** approximation on all simplicial components of CS

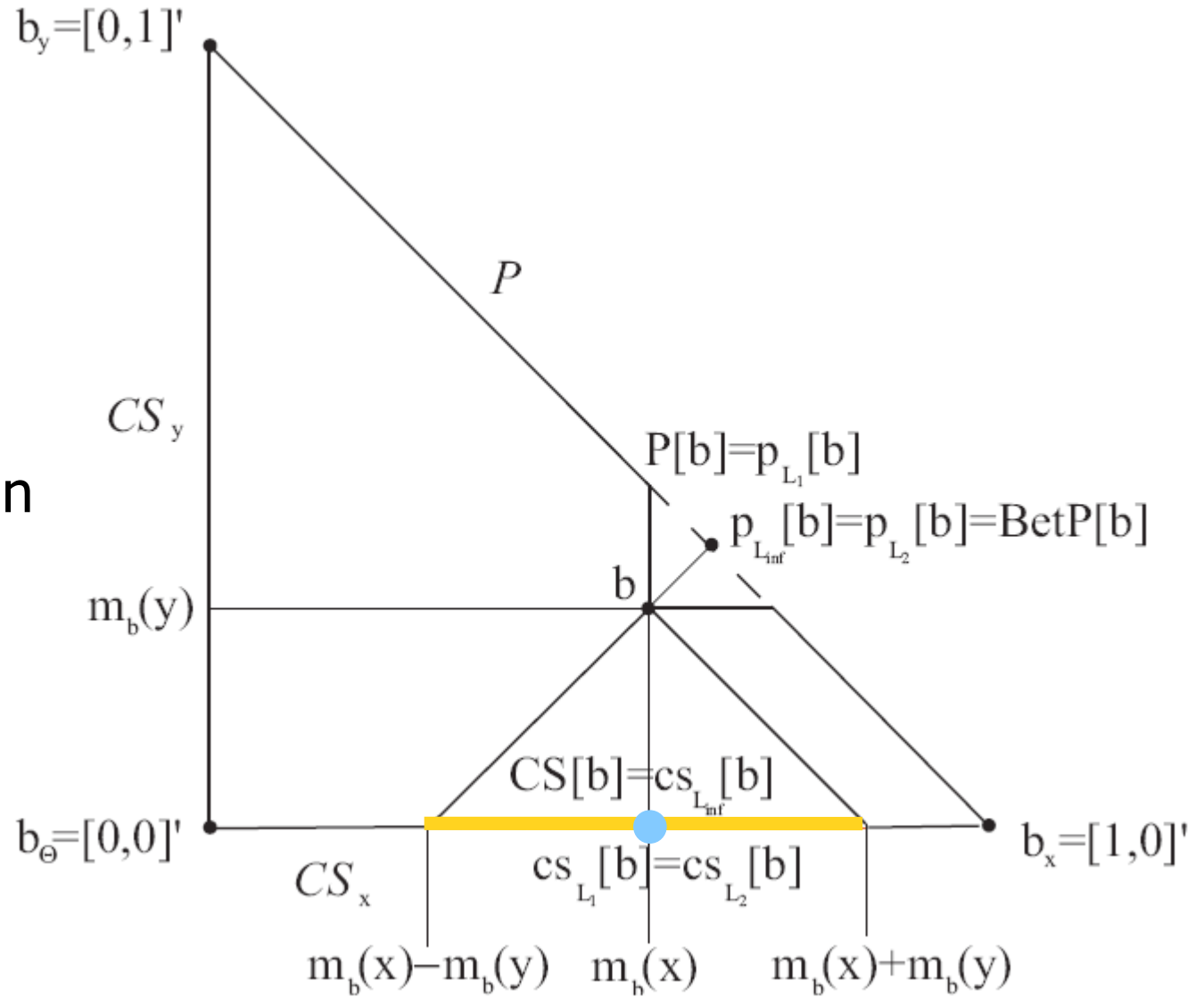


- **global** solution = best such approximation

# The binary case



- consistent approximation in the binary case





# Partial $L_p$ approximations

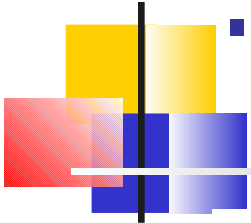
- $L_1 = L_2$  approximations have a simple interpretation **in terms of belief** [IEEE TFS07]



- left: a belief function      right: its consistent approx  
focused on x

$$m'(A \cup x) = m(A) \quad \forall A$$

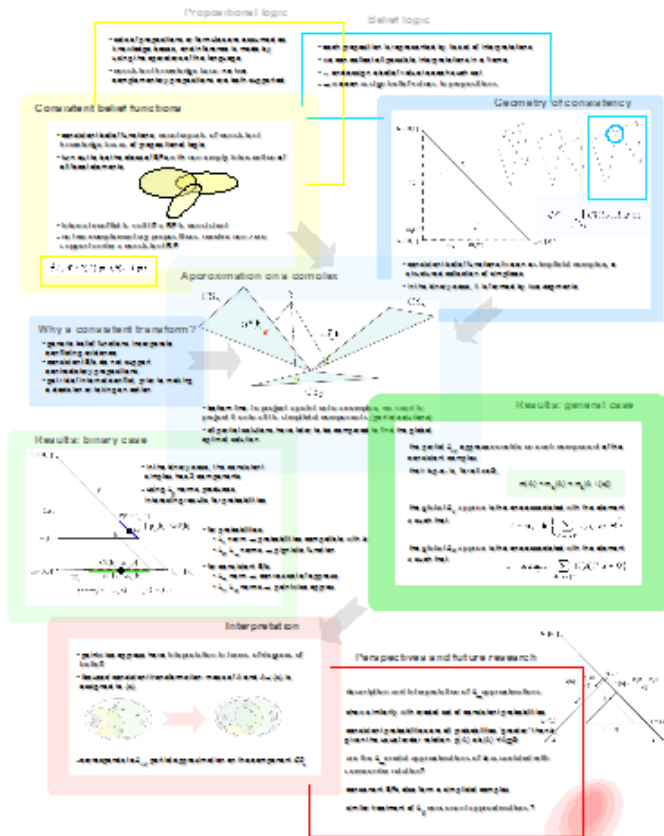
# ... please come to my posters!



8<sup>th</sup> INTERNATIONAL SYMPOSIUM ON IMPRECISE PROBABILITIES AND THEIR APPLICATIONS, BIFA'08  
 Oxford Brookes University, UK, July 14-18 2008

## Consistent approximations of belief functions

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## Credal semantics of Bayesian transformations

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