Nonparametric Predictive Multiple Comparisons with Censored Data and Competing Risks

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Nonparametric Predictive Inference (NPI)

- NPI is based on Hill's assumption $A_{(n)}$ (Hill, 1968), which implies direct (lower and upper) probabilities for a future observable random quantity, based on observed values of n related random quantities (Coolen, 2006).

- Coolen and Yan (2004) presented rc-$A_{(n)}$ as a generalization of $A_{(n)}$ for right-censored data.

- Coolen and Yan (2003) developed NPI for comparison of two groups of lifetime data including right-censored observations.

Early termination of experiment

- All units are placed simultaneously on a lifetime experiment which is terminated at a certain specified time, or when a specified number of failures have occurred. Known as Precedence testing (Balakrishnan & Ng, 2006)

- The lifetime experiment is ended early in order to save costs or time.

- We introduced NPI for comparison of two groups of lifetime data with early termination of the experiment, say at time T_0, and they illustrated the effect of varying T_0.

Cont. Early termination of experiment

- We extend this to more than two groups, with a variety of inferential goals for the multiple comparisons including, selecting the best group, the subset of best groups and the subset that consists the best group.

- We present further generalized results, which also generalize the results by Coolen and Yan (2003), by developing NPI for comparison of multiple groups of lifetime data including right-censored observations, and with possible early termination of the experiment.

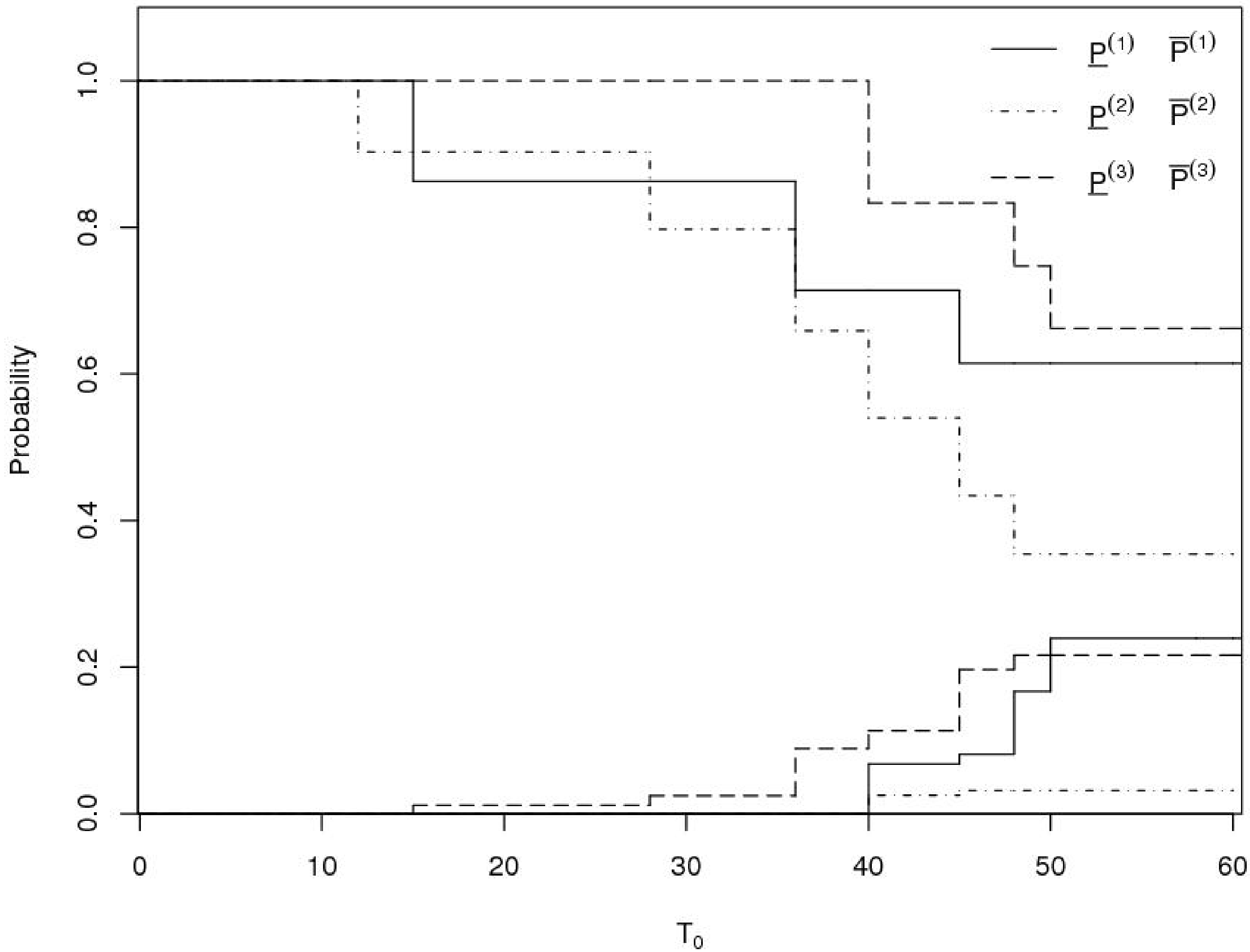
Example: Early termination

Desu and Raghavarao (2004) present recorded times (months) until promotion at a large company, for 19 employees in $k = 3$ departments.

Dept 1: $15, 20^+, 36, 45, 58, 60$
Dept 2: $12, 25^+, 28, 30^+, 30^+, 36, 40, 45, 48$
Dept 3: $30^+, 40, 48, 50$

where $+$ indicates that the employee left the company at that length of service before getting promotion.

At which department that one needs to work the longest to get a promotion??



$\underline{P}^{(1)}$ $\overline{P}^{(1)}$
$\underline{P}^{(2)}$ $\overline{P}^{(2)}$
$\underline{P}^{(3)}$ $\overline{P}^{(3)}$
Probability
0.0
0.2
0.4
0.6
0.8
1.0
0
10
20
30
40
50
60
T_0

Progressive censoring

- Some units are randomly removed from the experiment at several stages.

- For a progressive Type-II censoring scheme, R_i units are removed from the experiment at the ith failure.

- We introduce NPI for comparing two independent groups under several progressively censored schemes, including progressive Type-II, progressive Type-I and Type-II progressively hybrid censoring.

Competing risks

- A unit is subject to failure from one of k distinct failure modes (known and independent).

- We suppose that such failure observations are obtained for n units.

- For the NPI approach, let the failure time of a future item be denoted by X_{n+1}, and let $X_{j,n+1}$ be the failure time of an item due to failure mode j.

- We derived the NPI lower and upper probabilities for this event, $X_{n+1} = \min_{1 \leq j \leq k} X_{j,n+1}$

Example: Competing risks

36 units of a new model of a small electrical appliance. There were 18 failure modes (FM) in which an appliance could fail.(Lawless, 2003)

# cycles	FM	# cycles	FM	# cycles	FM	# cycles	FM
11	1	1167	9	2551	9	3112	9
35	15	1594	2	2565	-	3214	9
49	15	1925	9	2568	9	3478	9
170	6	1990	9	2702	10	3504	9
329	6	2223	9	2761	6	4329	9
381	6	2327	6	2831	2	6367	-
708	6	2400	9	3034	9	6976	9
958	10	2451	5	3034	9	7846	9
1062	5	2471	9	3059	6	13403	-

The NPI lower and upper probabilities for the event that unit 37 will fail due to FM9 are

$$\underline{P}(X_{37}^{FM9} < X_{37}^{OFM}) = 0.4358$$

$$\overline{P}(X_{37}^{FM9} < X_{37}^{OFM}) = 0.5804$$

The NPI lower and upper probabilities for the event that unit 37 will fail due to FM9, FM6 or OFM, are

$$\underline{P}\left(X_{37}^{FM9} < \min\left\{X_{37}^{FM6}, X_{37}^{OFM}\right\}\right) = 0.3915$$

$$\overline{P}\left(X_{37}^{FM9} < \min\left\{X_{37}^{FM6}, X_{37}^{OFM}\right\}\right) = 0.5804$$

$$\underline{P}\left(X_{37}^{FM6} < \min\left\{X_{37}^{FM9}, X_{37}^{OFM}\right\}\right) = 0.1749$$

$$\overline{P}\left(X_{37}^{FM6} < \min\left\{X_{37}^{FM9}, X_{37}^{OFM}\right\}\right) = 0.3279$$

$$\underline{P}\left(X_{37}^{OFM} < \min\left\{X_{37}^{FM6}, X_{37}^{FM9}\right\}\right) = 0.2265$$

$$\overline{P}\left(X_{37}^{OFM} < \min\left\{X_{37}^{FM6}, X_{37}^{FM9}\right\}\right) = 0.3808$$

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